# 9 Electron Charge to Mass Ratio

## 9.1 Summary

This experiment uses a kinematic analysis of the helical path followed by electrons deflected by a magnetic field to determine the ratio of their electrical charge $e$ to their mass $m_e$. The ratio is determined by measuring the change in magnetic field required to keep an electron beam focused on a CRT screen as a function of different accelerating voltages (also known as Hoag's method).

### Objectives

1. Develop experience with the motion of charged particles in electric and magnetic fields.
2. Measure the charge to mass ratio of the electron.
3. Estimate the magnitude of the Earth's magnetic field.

### Equipment

Army surplus type 902 cathode ray tube (CRT); Helmholtz coils to produce a nearly uniform magnetic field; high voltage power supply; hand compass; travelling microscope; X-ray photo of the CRT; 10 cm calibrating rod; multimeter.

### Principal Data Taken

1. Deflection voltage as a function of coil current; length of CRT tube.

### Things to Watch Out For

- The Helmholtz coils produce a very high magnetic field when the current is turned on. Keep delicate mechanical items such as analog watches away from the coils or you risk ruining them by magnetization of small components.
- The phosphor screen on the inside of the CRT is delicate and susceptible to damage caused by overheating. When the electron beam is too intense the screen risks being damaged. Always make sure to keep the brightness as low as possible; it will be useful to work with the room lights turned off if possible. One sign of temporary damage to the screen is the "black spot", in which the central region of the tube appears to burn out. This will make it very difficult if not impossible to take data until the accumulated charge diffuses away and the phosphor relaxes back to its original distribution (which can take a few minutes up to an hour).

## 9.2 Theoretical Background

The motion of electrons under the combined effects of an electrical field $\vec{E}$ and a magnetic field $\vec{B}$ is given by the Lorentz force, $\vec{F} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right)$, where $\vec{v}$ is the electron velocity and $e$ is its charge. Because the acceleration of the electron is given by F/m, relatively straightforward experiments using approximately constant magnitude electric and magnetic fields are expected to produce direct measurements of the ratio of the charge on the electron to its mass.

In this experiment electrons are accelerated by a cathode inside a vacuum tube (a CRT). This creates an electron beam that impacts a phosphor screen on the end of the tube, causing the phosphor to glow. The electron optics are controlled by means of potentiometers and additional anodes inside the tube that allow the glowing spot on the phosphor screen to be focused.

A low frequency (50 Hz), alternating voltage is applied to the deflector plates of the CRT, causing a stripe to appear on the screen instead of a best focus dot.

The electron beam is created travelling parallel to a nearly uniform magnetic field. The magnetic field is controlled by sending a high current through a large coil of wire. The field along the axis of a solenoid depende on the number of coils in the wire, the total length of the solenoid, and the current through the wires. As long as an electron is moving purely parallel to the magnetic field lines, its motion is helical - it travels at constant velocity along the field line, but circulates around the field line with a uniform angular frequency.

A complete derivation of the equation of motion, necessary to answer some of the theoretical questions, may be found in Appendix A at the end of this chapter.

As the magnetic field B is steadily increased from zero, the stripe rotates, and may shrink to an extremely short line (∼1 mm or less long) when a particular value of the field is achieved. At larger values of B the stripe expands and continues to rotate. For the minimum length condition, it is shown in Appendix A that

$$\frac{e}{m} = \frac{8\pi^2 V}{B^2 L^2}, \tag{9.1}$$

Where V is the potential difference through which the electrons are accelerated and L is the distance to the screen from the centre of the deflector plates ("Y-plates") of the CRT. B is not directly measured, but is a simple function of the current through the solenoid coils.

**Discuss the following in your report:**

After you've done the experiment, considered the equation of motion, and checked any necessary references, discuss why it is that the stripe on the screen remains straight as it rotates. This is a key feature of the equation of motion and illustrates your grasp of the 3-dimensional electron motion and its interaction with the CRT screen.

## 9.3 Procedure

1. Switch on the power supply and familiarise yourself with the functions of the various controls. Avoid leaving a bright trace on the screen for more than a second or two. The

cathode ray tubes we use in this experiment are very old. In fact they are ex-US army tubes, type 902, from World War II. We use them because the Y deflecting plates are simple rectangles, which make it easy to analyse the motion of electrons in the tube. More modern tubes use flared deflecting plates. However, the age of the tubes means that the phosphor on the screens is very susceptible to damage caused by local overheating, in turn caused by a too bright trace. So always make sure to keep the brightness as low as possible, and if you can do so without annoying other experimenters, work with subdued room lighting.

2. With zero coil current and zero deflection voltage, focus the spot by means of the focus control (use very low intensity).

3. Align the coils so that their axis is in the plane of the Earth's field. Use the compass provided. Then move the compass at least 3 metres away so that its magnetism is not subsequently permanently affected by the coils' magnetic field. Keep your wristwatch well away from the coils too, or it may be permanently magnetised.

4. Rotate the coils about a horizontal axis until maximum horizontal X deflection of the spot is observed. Read the angle associated with this orientation on the graduated scale. Now reverse the rotation and note the angle associated with maximum X deflection in the other direction. The two angles should differ by 180°.

5. Rotate the coils back to the midway position, which should be such that the coil axis makes an angle of about 72° with the horizontal. Now the axis should be parallel to the magnetic field of the Earth, $B_E$. You can check that the coil field $B_C$ is aligned with $B_E$ by carefully observing the spot while the coil current is rapidly increased. No deviation of the spot means that the axis of the CRT and the coil are both parallel to $B_E$. You may find that it is impossible to achieve less than a certain minimum deviation. This would be due to slight misalignment of the CRT axis with the coil axis. You should not attempt to correct this, because of the risk of damage to the tube.

6. Set the H.T. control on the blue console to about 750 volts. Reduce the brightness to a very low level. Increase the deflection voltage until a vertical stripe about 1.5 cm in length is produced on the screen. Set the three-position coil-current switch in the down position, which directs $B_C$ down the tube axis (away from the screen). Increase the coil current and observe the clockwise rotation and shrinkage of the stripe.

   With great care, observe the value of coil current associated with the 'best focus' condition, i.e., when the stripe is reduced to a very short line. Use a hand lens to view the screen and the meters.

7. Obtain a set of readings for different accelerating voltages V and the associated coil currents I. Plot $\sqrt{V}$ against I and check that it is a good straight line. Should it pass through the origin? Why not plot V against $I^2$? Calculate the slope of the graph and estimate the error. Do this carefully and in accordance with the principles laid out in Chapter 3.

8. Repeat the observations with the reverse direction of $B_C$ and again calculate the slope of the graph of $\sqrt{V}$ versus I. Plot this graph on the same axes as the previous one, for comparison. Are the two slopes statistically different? What about the intercepts?

9. With the aid of the travelling microscope, measure the important dimensions of the CRT on the X-ray negatives provided. Measure the 10 cm rod whose image is included in the pictures. Hence correct the CRT dimension measurements for X-ray projection error and

film shrinkage. Calculate L and estimate the error. The thickness of the screen glass is best measured directly from the fragments of a similar tube, which are available for measurement with calipers. Accuracy here is critical because of the dependence of the results on the value of L.

10. Reduce the coil current and the voltage to very low levels and shut everything down.

## 9.4 Calculations

For the Helmholtz coils used in this experiment, the axial magnetic field is linearly related to the coil current I:

$$B_C = kI, \tag{9.2}$$

where the coil constant $k = 161.3$ Gauss·Amp$^{-1}$, i.e., $k = 1.613\times10^{-2}$ T·A$^{-1}$. The uncertainty in this figure is $\pm0.5\%$.

Starting from Equation 9.1, show that if $S$ is the slope of a graph of $\sqrt{V}$ against I, then

$$\frac{e}{m} = \frac{8\pi^2 S^2}{k^2 L^2}, \tag{9.3}$$

1. By carefully considering the effect of all the errors, you should be able to obtain a high-precision value of $e$/m that is consistent with the accepted value of $1.76\times10^{11}$ C·kg$^{-1}$. Make sure to report all your measurements with errors, as well as the slopes of your graphs with their errors, and the errors in your final calculated quantities. Which quantity most strongly contributed to the final uncertainty in $e$/m?
2. Why didn't you plot V versus $I^2$ in your graphs?
3. If your work has been of sufficient precision, you will also be able to give an estimate for the magnitude of $B_E$, the Earth's magnetic flux density inside the laboratory. Explain clearly how this relates to your measurements and your graphs.

**References**

Hoag, J.B. *Electron and Nuclear Physics*, p. 32

Watson, R.D. 1998, *"Grey Binder": e/m Ratio of Electrons* (not to be removed from the lab).

# Appendix A: Electron motion in the $e/m$ apparatus

Consider an electron initially travelling along the axis of the tube. A schematic representation is shown in Figure .1. The magnetic field is parallel to the axis of the tube. After leaving the cathode the electron experiences no forces until it enters the space between the Y-deflector plates (Region 1). While is between the plates, during a time of about 1 ns, the force on the electron is given by

$$F_1 = -e\left[\vec{E} + (\vec{v} \times \vec{B})\right] \tag{.1}$$

Thereafter, in Region 2, only the magnetic field is acting and the force on the electron is

$$F_2 = -e\left(\vec{v} \times \vec{B}\right) \tag{.2}$$

For the coordinate system illustrated in Figure .1, the various vectors are specified as indicated in Table .1.

Some basic kinematics will show you that the magnitude of the acceleration $a$ = $e$E/m, and the angular frequency of the motion is $\omega$ = $e$B/m.

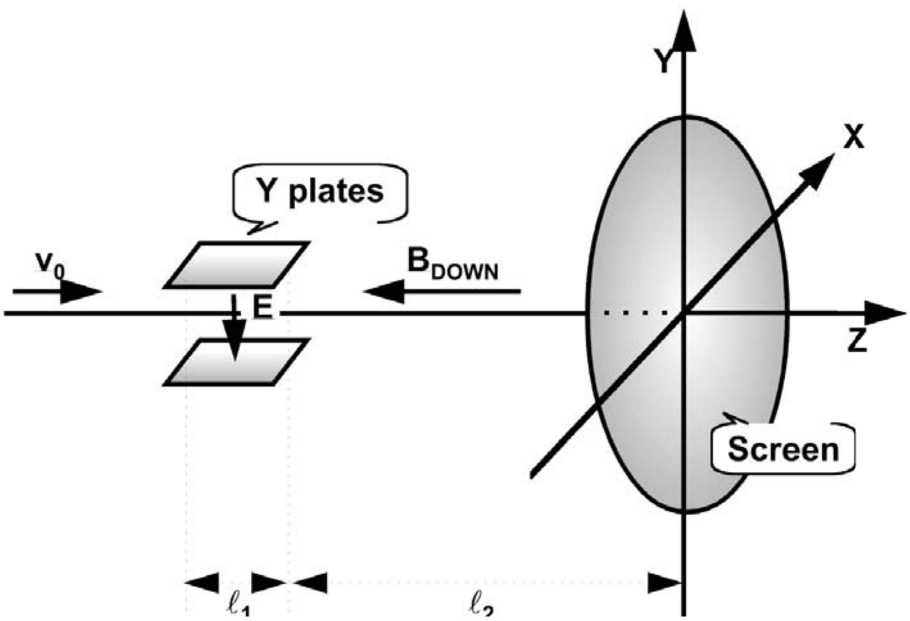


Figure .1: Schematic of the deflector plates and screen showing the forces acting on the electron.

Manipulation of Equations .1 and .2 yield the following differential equations for Region 1:

$$\ddot{x} - \omega\dot{y} = 0 \quad (.3)$$
$$\ddot{y} + \omega\dot{x} = a \quad (.4)$$
$$\ddot{z} = 0 \quad (.5)$$

For the boundary conditions $x(0) = 0$, $y(0) = 0$, $z(0) = z_0$, and $v(\vec{0}) = \vec{v_0}$, the solutions to these equations are:

$$x = \frac{a}{\omega^2}(\omega t - \sin\omega t) \quad (.6)$$
$$y = \frac{a}{\omega^2}(1 - \cos\omega t) \quad (.7)$$
$$z = (z_0 + v_0 t) \quad (.8)$$

These equations give motion at constant velocity in the $z$-direction, but note that the $x$ and $y$ motion are oscillatory. Taken together, the $x$ and $y$ equations are the parametric specification of the electron path, projected onto the $xy$ plane. The form of the equation is a cycloid – this is geometrically equivalent to the curve traced out by a point on the circumference of a circle as the circle rolls along a straight line (shown in Figure .2).

From this we see that in Region 1 the electron moves away from the $x$-axis at a variable speed, and oscillates in the $y$-direction between 0 and some maximum value. This is equivalent to a circular motion around a fixed centre that moves steadily in the $x$-direction at a speed given by $u = (\mathrm{R}\omega - \mathrm{E}/\mathrm{B})$, where R is a constant that has geometric meaning as the radius of the notional circle rolling along the $x$-axis. This cycloidal motion has the effect of reducing the angular velocity of the electron by half, to $\omega/2$, accounting for an error in Hoag's original experiment.

When the electron moves into Region 2, the equations of motion become:

$$\ddot{x} - \omega\dot{y} = 0 \quad (.9)$$
$$\ddot{y} + \omega\dot{x} = 0 \quad (.10)$$
$$\ddot{z} = 0 \quad (.11)$$

which has a simpler solution. It is left to the student to show that in Region 2, the motion is helical (see Figure .3). The figure shows that in the idealised focus condition, the electron

Table .1: Vector quantities in the coordinate system of Fig. .1.

| | |
|---|---|
| Initial velocity | $\vec{v_0} = (0, 0, \mathrm{v}_0)$ |
| Electric Field (Region 1) | $\vec{E} = (0, -\mathrm{E}, 0)$ |
| Magnetic Field | $\vec{B} = (0, 0, -\mathrm{B})$ |
| Electron velocity | $\vec{v} = (\dot{x}, \dot{y}, \dot{z})$ |

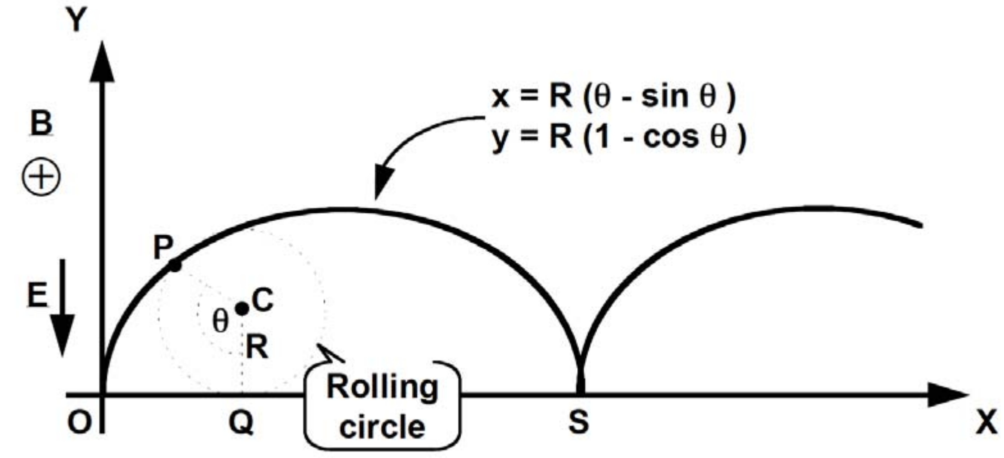


Figure .2: Graphic representation of the electron motion in the $xy$ plane, when it is subject to a constant electric field in the negative $y$ direction (i.e., while it is in Region 1).

would return to its initial position at the origin at exactly the same time as it arrived at the CRT screen; due to the deflection by the electric field in Region 1 and the consequent initial cycloid motion, this is not possible.

As the focus is adjusted, the potential difference between the cathode and the screen is adjusted such that the electrons striking the screen are located at different points along the helix shown in Figure .3. The initial velocity, strength of the magnetic field, and charge to mass ratio of the electron all play a role in the radius and frequency of the helical path taken.

If $t_1$ is the time of transit of Region 1, and $t_2$ is the time of transit of Region 2, then the condition of best focus requires that

$$\phi + \omega t_2 = 2\pi \tag{.12}$$

$\phi$ is the helix angle upon entering Region 2, and from the diagrams we can see that it must be related to the cycloid angle $\theta = \omega t$. In fact it can be shown that $\phi = \omega/2$, so Equation .12 becomes

$$\omega\left(\frac{t_1}{2} + t_2\right) = 2\pi \tag{.13}$$

Because time = distance divided by velocity, and $v$ is constant, we can replace this by

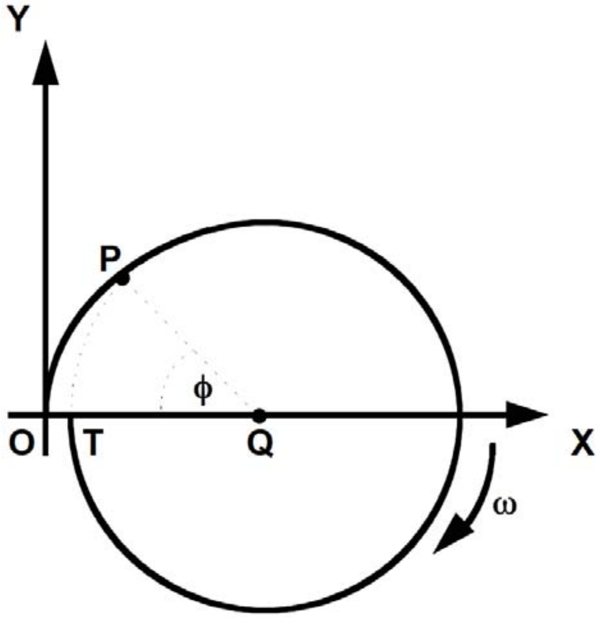


Figure .3: Face-on view of the electron motion in Region 2 of the apparatus (no electric field). If the electron leaves Region 1 at point P, it can be shown that the motion in Region 2 is a helix around the fixed central axis at point Q. Clearly the condition of best focus is achieved when the electron hits the CRT screen at point T.

$$\omega \left( \frac{l_1}{2} + l_2 \right) / v_0 = 2\pi \tag{.14}$$

A kinematic analysis of the electron acceleration by the CRT electrodes allows us to derive the initial velocity $v_0$ as a function of the operating voltage V:

$$v_0 = \sqrt{\frac{2eV}{m}} \tag{.15}$$

If we use the definition of $\omega$ in terms of B, e, and m from above, then the equation of best focus becomes

$$\frac{B \frac{e}{m} \left( \frac{l_1}{2} + l_2 \right)}{\sqrt{\frac{2eV}{m}}} = 2\pi \tag{.16}$$

which can be reduced to a simple expression for the charge to mass ratio of the electon,

$$\frac{e}{m} = \frac{8\pi^2 V}{B^2 L^2} \tag{.17}$$

where the effective length L $= \frac{l_1}{2} + l_2$.

The equations of motion may be explicitly solved for $t \neq 0$ to yield expressions for the $x$ and $y$ position at some arbitrary time $t_1$. These are the steps you would take in order to predict the location of the spot on the CRT screen as a function of time. Additional notes contained in the grey binder notebook show the derivation in full.