6 The Earth's Magnetic Field

6.1 Summary

A changing magnetic flux creates a measurable electric current in a loop of wire. One way of creating a change in magnetic flux is by changing the orientation of the loop with respect to a constant field $\vec{B}$. In this experiment, the strength and orientation of the Earth's magnetic field is measured by observing the electrical response produced during a 180° rotation of a coil of wire (known as the "π-flipper"). Because the Earth's magnetic field is relatively weak, an op-amp circuit is used to amplify the signal.

Objectives

1. To gain practical experience applying Faraday's laws of induction.

2. To measure the vertical and horizontal components of the Earth's magnetic field.

3. To work with operational amplifier circuitry ("op-amps").

Equipment

Spring-loaded "π-flipper" coil in wooden frame, integrating amplifier "black box" with adjusting screwdriver, digital voltmeter and/or oscilloscope, battery pack, adjustable leveling stand, spirit level, compass.

Principal Data Taken

1. Integrated changes in voltage when coil flips, measured indoors and outdoors.

2. Dimensions, orientations, and resistance of the π-flipper.

3. Electrical properties of the components in the integrating amplifier.

Things to Watch Out For

- Don't apply an input voltage to the amplifier until its own power supply has been connected up and turned on, or the chip inside may be fried. If you're convinced you have a good setup, and still get bad results, the integrated circuit chip may need to be replaced; see your demonstrator.

- All voltage supplies, meters, and amplifiers must be connected to a common earth, or the amplifier will give nonsensical or misleading results. One symptom may be that you are unable to zero the voltage produced by the amplifier. Take care not to subject the π-flipper or the amplifier box to shocks or drops.

6.2 Theoretical Background

Prior to the 17th century the deflection of a compass needle to the north was explained by postulating that either there were massive deposits of lodestone (magnetite, Fe_3O_4) at the north pole, or that the needle was attracted by some (quasi-mystical) property of the star Polaris (α Ursa Minoris). In 1600, William Gilbert published his work "De Magnete", in which he advanced the hypothesis that the Earth itself generated its own magnetic field, and that it had a similar configuration to a magnetized sphere. By observing the orientation of small needles placed at various spots around such a sphere, he verified that his hypothesis was able to explain much of the mysterious behaviour of real compasses.

In modern terms, the Earth behaves to first order like a magnetic dipole whose poles are slightly offset from the north and south geographic poles. This results in a magnetic field on the Earth's surface, B_E, which varies in strength and direction from place to place. An important observable property of B_E is that it has a vertical component, known as the dip, that increases with geomagnetic latitude. Another observable is the difference between magnetic and geographic north (or south). This difference is termed the magnetic variation or magnetic declination. Both angles are observed to vary with time, and the rate and periodicity of this "secular variation" provides important clues to the processes which create the Earth's magnetic field.

Changes in magnetic fields can be measured by their effects on conducting materials. In particular, a closed loop of wire interacts with a changing magnetic field to produce a voltage (and an associated current) through electromagnetic induction. In this context, the voltage created is known for historical reasons as "electromotive force" (EMF, or $\mathcal{E}$), although it is not a force! Faraday's law states that the EMF in a single conducting loop is given by the rate of change of magnetic flux,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}.$$

If the loop is made up of a large number of differential area elements $d\vec{A}$, the magnetic flux Φ_B is the dot product $\vec{B} \cdot d\vec{A}$, integrated over the total loop area. The flux can change either due to change in the strength or direction of the magnetic field, or due to a change in the size or orientation of the conducting coil.

Discuss the following in your report:

Consider a flat, circular coil of n turns, each of the same radius r, in a uniform magnetic field $\vec{B}$. The coil is placed perpendicular to $\vec{B}$, i.e., the unit vector $\vec{\hat{A}}$ normal to the circular area enclosed by the coil is parallel to $\vec{B}$. Now suppose the coil is rotated through an angle θ during time τ at a constant angular velocity, so that $\vec{\hat{A}}$ ends up anti-parallel to $\vec{B}$. During the rotation, a voltage $\mathcal{E}$ is induced across the coil. Once the rotation stops, the voltage drops back to zero. Starting from Faraday's law of induction, show that for $\theta = 180°$

$$\int_0^\tau \mathcal{E} dt = 2\pi n r^2 B \tag{6.1}$$

Now consider the case for which the the coil is *not* perpendicular to $\vec{B}$, i.e., $\vec{\hat{A}}$ is at an angle ϕ to $\vec{B}$. What is the physical interpretation of the quantity $\int_0^\tau \mathcal{E} dt$?

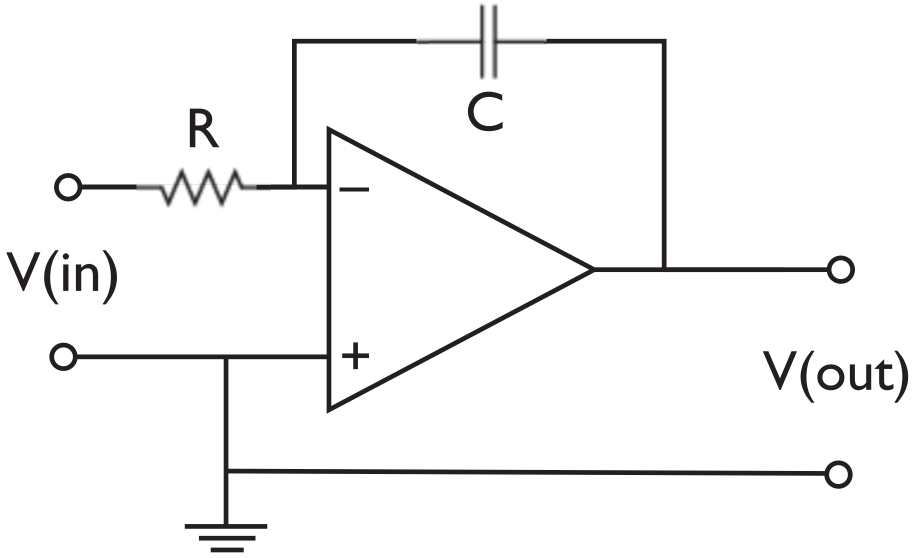

Figure 6.1: Schematic of an integrating amplifier that accepts input voltage V_{in}, amplifies it by a factor 1/RC, and shows the integrated output across V_{out}. The power supply, reset button, drift adjustment, and additional resistive and capacitive components that protect the circuit are not shown.

The amplifier circuit will be used to measure the EMF produced when the coil flips. It has an operational amplifier (op-amp, circuit symbol a triangle with two *in* and one *out* ports), a resistor R and a capacitor C. In the ideal case, with no additional resistance or inductive losses, the amplification factor is given by

$$Gain = \frac{1}{RC} \tag{6.2}$$

An ideal op-amp keeps the voltage difference between the input pins zero, and the input draws no current. Consider the schematic circuit diagram in Figure 6.1 together with Equation 6.2. In your report, applying Ohm's Law and the definition of capacitance to the input and output sides of the circuit, derive the relationship between the input and output voltages.

6.3 Procedure

The π-flipper coil is a length of wire with ≈1000 turns with mean diameter of about 15.5 cm. It is mounted on a spring-loaded shaft attached to a wooden frame on a tripod, which can be wound up and released with a simple ratchet. When released, the coil rotates through an angle of 180° (i.e., π radians, hence the name), in a time of about 0.2 seconds. The ratchet mechansim locks at the end of a flip to prevent bouncing.

The π-flipper can be considered a very primitive flux gate compass. A modern navigational flux gate compass uses the same principles to measure an electrical output induced by the Earth's magnetic field, but uses three orthogonal coils wrapped on a highly ferromagnetic core, subject to an applied AC voltage (so that the compass produces a reading without having to constantly rotate in three axes).

Local inductive effects (called deviation) can have strong impact on the deflection of a compass needle, and it is important to compensate or minimize these in order to obtain an accurate reading. In a steel ship, the deviation can be on the order of 90°, and depends on the orientation of the ship. If you have ever observed the compass in the instrument panel of a light airplane or helicopter swing through tens of degrees when the engine is started, you will understand the importance and difficulty of proper compass compensation.

You will measure the influence of local effects on the magnetic field by taking data from the π-flipper apparatus both inside and outside the lab (for example, in the grassy area just outside the 2nd floor entrance to the Physics building).

1. When the coil flips, a voltage is generated. This can easily be seen by connecting the leads of the voltmeter to the coil and flipping it, or moving it back and forth by hand. Two facts are important for the experimental procedure here– first, the voltages are quite small, on the order of millivolts or less; and second, the voltage returns to zero when the coil is not in motion. Try to record the raw voltages produced during a flip.

2. Equation 6.1 predicts that the component of $\vec{B}$ parallel to $\vec{A}$ can be measured by integrating the EMF over the time it takes the coil to flip. It is this integrated voltage that we wish to record on the voltmeter or oscilloscope. One approach could be graphical, taking a snapshot of an oscilloscope trace and integrating the area under the voltage vs. time curve. However, it will save work in the long run if we use an electronic integrator to do the job for us. This has the advantage that we can simultaneously integrate and amplify the voltage. The amplifier will yield voltages on the order of 0.1–1 V, which is easily measured with a standard voltmeter.

   An integrating amplifier is provided in the "black box"– one side of the unit has been removed so that you can see the components. The integrating circuit is shown schematically in Figure 1. It is build around an LM301 integrated circuit chip. This is a very common, cheap, and powerful device known as an operational amplifier, or "op-amp". By using various resistors, capacitors, and inductors conducted to the ports of the op-amp chip, it can be made to multiply, divide, integrate, or differentiate applied signals.

   More information can be found consulting the references at the end of this section, and the notes in the manilla folder on the bench near this experiment (National Semiconductor LM301 Reference Sheet). In brief, the LM301 has 8 pins that either accept input signals or produce output.

3. Two of the pins are connected to the black box + and − ports. These are located on the end of the box with the label marked "IN". These are the signal inputs to be connected to the π-flipper coil. A third port just below the inputs should be connected to a local earth.

4. A third pin of the LM301 outputs the *difference* between the two input channels. This difference is amplified by the circuitry attached to the op-amp. The amplification depends on the resistance and capacitance of the attached circuit and is typically on the order of a factor of 10^2–10^4. Connect the black box output ports to a voltmeter.

5. Amplification of a signal requires external power. A positive and negative voltage applied to the appropriate pins of the LM301 provide a power on the order of 10–20 volts. The supplied battery pack allows you to connect a supply of +V to the V_+ port, and −V to the V_- port. Note that the battery pack (or DC power supply if used) must be earthed locally with the inputs in order to provide a common ground potential.

The maximum output of the amplifier is determined by the voltage supplied. Because of the high amplification factor, tiny differences across the input terminals typically produce large output voltages, so it is common to see the output very quickly rise to within a fraction of a volt of the power supply voltage, at which point the amplifier is said to be "saturated". It typically takes an input voltage differential of only a few μV to saturate an op-amp being run off a 12-V power supply.

An integrating amplifier relies on a feedback loop, whereby the output channel from the op-amp chip is fed back into one of the input channels. The size of the feedback loop is controlled by adding resistors and capacitors into the loop. By choosing what circuit elements to use, various operations can be carried out, such as differentiation or integration. Much more information can be found in the Kuphaldt reference listed at the end of this section. The circuit diagram shown in Figure 6.1 gives a simplified view of an integrator.

At any given time, the voltage that appears across the output terminals of the integrating amplifier shows the integral of the voltage applied over time. One source of voltage will be the signal you are interested in, but other sources including local inductive effects and thermal noise are also present. This will cause the output voltage to rapidly drift (either positive or negative) until it reaches the saturation point.

6. There is a reset button to null the output, and a port for a screwdriver adjustment to an internal potentiometer. This potentiometer can be adjusted in order to null the amplifier drift rate. Keep adjusting the potentiometer until the output only changes very slowly, or not at all. It doesn't have to be nulled out at zero – once the voltage is constant you can set it to zero with the reset button. Note that in all measurements, the *sign* of the output voltage is an arbitrary quantity because it depends on the wiring details and the direction of the coil flip.

7. The component values in the amplifier box were chosen to provide a relatively easy-to-measure output voltage, because the builder knew the approximate magnitude of B_E and worked out the expected value of $\mathcal{E}$ from Equation 6.1. The chosen values are R = 22±2.2 kΩ and C = 0.10±0.01 μF. All of the other extra components are included to protect the circuit from overload and to counteract non-ideal effects. Calculate the expected gain of the amplifier.

8. During the coil flip, does the drift in the amplifier contribute significantly to your measurement uncertainty? What other resistive effects or other losses must be considered? Record any additional measurements you may need to make to find an effective resistance for the setup. In all cases, internal resistance in the voltmeter may be neglected.

9. Is the integrator repeatable? Test it out thoroughly for repeatability. Is it accurate? What factor limits the uncertainty in your output voltages? Can you think of any ways to decrease this uncertainty with an additional measurement or set of measurements?

10. Measure the horizontal (H) and vertical (Z) components of B_E with the coil on the lab bench. Use the adjustable stand and the spirit level to level the coil when making the Z measurement. A compass is provided for finding magnetic north. How can you be sure you are measuring the total H component of the field? What should V_{out} be when the coil is facing magnetic east or west? Do your measurements agree? Describe any steps taken to reduce the errors in coil alignment, and estimate the uncertainty with which you can measure the alignment angle. Take enough measurements so you will be able to calculate a standard deviation for your results.

11. Repeat all your measurements from the previous step in an interference-free environment. Try taking the setup outside, far from large metal objects or electrical power lines. Don't leave any equipment outside!

6.4 Calculations

1. Using your data, calculate Z, H, and the dip angle (the angle between B_E and horizontal, increasing downward) at Hobart. Compare your values to the International Geomagnetic Reference Field (see References below). Include all the uncertainties and show your propagation of errors.

2. Compare your indoor and outdoor measurements. Do they agree within errors? Identify what you think are the major likely sources of deviation (local inductive elements).

3. Why is the measurement of H so sensitive to errors in coil alignment while Z is much less so? Are there locations on the Earth's surface where this situation would be reversed? Is there a minimum or maximum distance from a geomagnetic pole beyond which the procedure here becomes completely unable to provide a reliable measurement of H? What steps could you take to circumvent this problem (apart from buying expensive, professional grade navigational or geomagnetic survey equipment).

References

The induction section from any E&M or general physics textbook, including (for example), Halliday, Resnick & Walker, "Fundamentals of Physics", or D.J. Griffiths, "Introduction to Electrodynamics"

"International Geomagnetic Reference Field – 11", numerous versions available online– e.g., `http://wdc.kugi.kyoto-u.ac.jp/igrf/`

"Introduction: Operational Amplifiers", edited by T.R. Kuphaldt et al. (1996–2013), `http://www.allaboutcircuits.com/vol_3/chpt_8/1.html`.