UNIVERSITY *of*
TASMANIA

PART II LABORATORY WORK

KYA 211/212

Ruchardt's Measurement of γ

Last compiled 2026-04-02

Safety

This experiment requires the use of gasses which are stored in high-pressure cylinders. Due care should be taken when working with compressed gas; ensure that you are confident in the operation of a regulator prior to making any adjustments, and only make small adjustments. Instructions on using gas regulators are found in the appendix. Caution should also be exercised when operating the Rüchardt apparatus, as it is possible to launch the bearing ball at high velocity from the oscillation tube.

Attaining values of the ambient pressure requires the use of a Mercury barometer. Care should be taken to not damage the glass tube of the barometer.

Outline

Summary

The period of a ball bearing undergoing lightly damped, forced oscillations caused by pressure variations in a gas reservoir is used to evaluate the adiabatic index, γ, of the gas. γ is a fundamental thermodynamic property that, according to theory, takes values equal to the ratio of two integers for monoatomic, diatomic, and triatomic gases. Examples include argon (monoatomic), dry air (a mixture of O_2 and N_2, diatomic), and CO_2 (triatomic). The adiabatic index γ is closely related to the gas's compressibility, speed of sound, and heat capacity.

Experiment Objectives

- **Primary:** Use kinetic theory calculations to measure the adiabatic index γ of argon, air, and CO_2 from oscillation timing data.
- **Secondary:** Verify that the frequency of oscillation of a pressurised gas system varies according to whether the gas is monoatomic, diatomic, or triatomic.

Pre-lab Exercises

Pre-lab questions should be completed and submitted before you commence a new experiment. The information needed to complete the exercises is contained in the Background Theory Section, your course notes, or in the Appendices however, your own, independent research is highly encouraged. Make sure you include references where material has been sought elsewhere. This is not only "good form" but making notes of important information is essential if you then need to go back to that reference.

Task

1. What is an *adiabatic* process? How does it compare to an *isothermal* process, an *isochoric/isometric* process, and an *isobaric* process? Describe how you would go about creating an adiabatic process, an isothermal process, an isochoric/isometric process, and an *isobaric* process?
2. Using equation 3, confirm Mayer's relation for an ideal gas, namely:

$$C_P - C_V = nR$$

   where n is the amount of substance and R is the ideal gas constant.
3. Consider the apparatus in Fig. 1. As described in the Background Theory Section on the Rüchardt experiment, when the ball is in equilibrium, the pressure in the system is given by $P = P_o + \frac{mg}{A}$ where P_o is the atmospheric pressure, g is the acceleration due to gravity, and m and A are the mass and cross-sectional area of the ball, respectively. If the ball is displaced a distance dy from its equilibrium position, it will start to oscillate. Starting from Equation 4, write out the full differential equation of motion (i.e. an expression for $\ddot{y}$) and show that it describes a damped oscillator with period T given by Equation 7.
4. The article in the appendix from Koehler 1951 describes a modification to the original Rüchardt apparatus. This modification is a small hole bored into the glass tube. What purpose does this modification serve and why was it necessary?

Background Theory

Adiabatic Processes

An adiabatic process is a type of thermodynamic process where no heat is exchanged with the surroundings ($Q = 0$), meaning any change in internal energy will be entirely due to work being done on or by the gas. In this case, the first law of thermodynamics becomes $\Delta U = -W$. Adiabatic processes occur rapidly, preventing heat transfer. For an ideal gas undergoing an adiabatic process, pressure and volume are related by the equation:

$$PV^{\gamma} = k$$

where γ is known as the adiabatic index and k is a constant. We will talk more about the adiabatic index below. An every-day example of an adiabatic process is the rapid compression of the petrol-air mixture in the cylinders of an internal combustion engine (such as the one likely found in your car... unless you drive an electric car... which is very good!).

The Adiabatic Index and Specific Heats

The adiabatic index γ is a fundamental thermodynamic quantity that describes how a gas responds to compression or expansion when no heat is exchanged with the surroundings. It is defined as the ratio of **specific heat capacities**,

$$\gamma = \frac{C_P}{C_V}$$

The specific heat capacities tell us about the amount of heat required to raise the temperature of one unit of mass by one degree. For gases, we define two specific heat capacities; the specific heat at **constant volume** C_V and the specific heat at **constant pressure**, C_P. The reason as to why is explained further down.

$$C_V = \frac{1}{m}\left(\frac{dQ}{dT}\right)_V \tag{1}$$

$$C_P = \frac{1}{m}\left(\frac{dQ}{dT}\right)_P \tag{2}$$

where dQ is the amount of heat required to raise the temperature of a mass m of gas by an amount dT at a constant pressure, P or a constant volume V.

The values of the specific heat capacities, (and thus, the value of of γ) are linked to the structural complexity of the gas and will change depending on whether the gas is monatomic, diatomic, triatomic... etc. This is because every gas molecule has different ways it can store energy (different degrees of freedom) and these include;

- Translational motion: Moving in the x, y, and z directions (all gases have this).
- Rotational motion: Spinning around different axes (this applies only for molecules, not single atoms).
- Vibrational motion: Stretching, compressing, flexing and twisting of the bonds between atoms in the molecule. At room temperature, many vibrational degrees of freedom do not contribute to a molecule's thermal energy. These degrees of freedom are "frozen out". Hence, air molecules (N_2 and O_2) have five degrees of freedom at

> room temperature and not seven. The explanation for why this is lies in quantum mechanics and is beyond the scope of our present study.

The more degrees of freedom a molecule has, the higher it's specific heat capacity will be. A smaller value of the specific heat capacity means a material will heat up quicker. As an example, if I were to demand that you hold a cup of boiling water but gave you the choice of either a metal cup or plastic cup, which would you choose? Steel has a specific heat capacity of approximately $0.45\,\mathrm{J\ g^{-1}K^{-1}}$, whereas plastic has a specific heat capacity of roughly $2.3\,\mathrm{J\ g^{-1}K^{-1}}$, meaning the metal cup will heat up much more rapidly than the plastic cup. Famously, it takes an enormous amount of energy to raise the temperature of water ($4.18\,\mathrm{J\ g^{-1}K^{-1}}$) but much less energy to raise the temperature of ice ($2.05\,\mathrm{J\ g^{-1}K^{-1}}$), which tells us that the specific heat capacity can be different for different phases of a substance. Actually, it turns out that it can vary wildly as a function of both pressure and temperature hence why we define specific heat capacities at constant pressure and at constant volume.

Standard values for γ for monatomic, diatomic and triatomic gases are given below.

$$\gamma_{\text{monatomic}} = 5/3 = 1.67$$
$$\gamma_{\text{diatomic}} = 7/5 = 1.40$$
$$\gamma_{\text{triatomic}} = 8/6 = 1.33$$

In the case of a **reversible** process, beginning with the second law of thermodynamics, it can be shown that the difference between heat capacities is related to the thermal expansion coefficient α and isothermal compressibility β_T via

$$C_P - C_V = VT\frac{\alpha^2}{\beta_T} \tag{3}$$

where

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$
$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

and the derivatives are obtained by differentiating the ideal gas equation. It can be shown that the above simplifies to $C_P - C_V = nR$ for an ideal gas (which you are asked to show in the prelab exercises). This is known as Mayers Relation and it outlines that C_P is always greater than C_V. This is because, at constant volume, any heat input goes straight into increasing the temperature of the gas. However, at constant pressure, the gas expands with heat input, meaning some of the heat goes into doing work on the surroundings rather than just increasing the temperature.

The Ruchardt Experiment

In this experiment, you will be using a modified Rüchardt apparatus to obtain values for the adiabatic index of various gasses. The Rüchardt experiment was first developed by German physicist Eduard Rüchardt and is now commonplace among thermodynamic experiments. One can determine the adiabatic index of a gas by analysing the oscillatory motion of a ball (or piston) in a vertical tube connected to a sealed gas-filled container. A schematic of a basic Rüchardt appartus is shown in Fig. 1. It's perhaps not an intuitive result that the adiabatic index (the ratio of specific heats) can be found from oscillatory motion but we shall outline the mathematical formalism below. In the prelab exercises you are asked to complete the derivation.

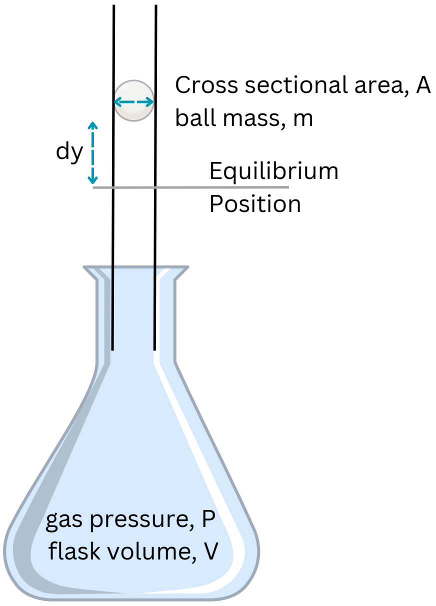


Figure 1: A schematic of a Ruchardt apparatus consisting of a single gas reservoir, a glass tube and a ball bearing that is free to move up and down inside the tube.

If we consider a system such as that in Fig. 1, the pressure in the system supports the ball and prevents it from falling to the bottom of the tube (in reality, there will be some leakage and obtaining perfect equilibrium will be difficult). When the ball is in equilibrium, the total pressure in the system is

$$P = P_o + \frac{mg}{A} \tag{4}$$

where P_o is the atmospheric pressure, m is the mass of the ball and A is the cross-sectional area of the ball.

If the ball is displaced some distance dy from equilibrium, the volume and pressure will change by

$$\begin{aligned} dV &= dyA \\ dP &= F/A \end{aligned}$$

where F is the restoring force acting on the ball. As long as the displacement occurs on a timescale that is small compared to the characteristic time for heat flow into and out of the gas, the condition for adiabatic compression/expansion is nearly satisfied, and $PV^\gamma = k$ can be assumed. We can differentiate this relationship to connect the adiabatic equation to the displacement relationships above,

$$\gamma PV^{\gamma-1}dV + V^\gamma dP = 0 \tag{5}$$

and by substituting in for dP and dV (as defined above),

$$F = -\frac{\gamma PA^2}{V}dy \tag{6}$$

This is the equation of a force acting on the ball that is linearly proportional to the displacement dy and directed towards the equilibrium position. The corresponding equation of motion describes a system undergoing simple harmonic oscillation. Because of frictional losses that lightly damp the system, the true equation of motion is slightly more complicated; in this experiment the light damping is overcome by a forcing term, namely the constant flow of gas into the system to replace that which is lost.

Fortunately, the period of a constant amplitude, forced oscillation is almost negligibly different from the undamped period as long as the damping is light and will have the form,

$$T \approx 2\pi \left(\frac{mV}{\gamma P A^2} \right)^{1/2} \tag{7}$$

By rearranging equation 7, we obtain the necessary equation for γ,

$$\gamma = \frac{4\pi^2 mV}{T^2 P A^2} \tag{8}$$

Apparatus

The apparatus you have for this experiment is a modified Rüchardt apparatus that allows for variable volume by having multiple connected gas vessels. A labelled sketch of the apparatus is given in Fig. 2.

The key components are:

- Gas cylinders containing argon and carbon dioxide gas, and an air compressor. The Argon and CO_2 cylinders are fitted with regulators. Notes on how to operate these are in the appendix.
- Modified Rüchardt apparatus - 6 glass bottles connected by a manifold to a vertical tube in which the oscillations may be observed.
- Stopwatches
- Barometer for taking atmospheric pressure readings (notes on reading the barometer are located in the appendix)

There are several already measured quantities which you will need for your calculations during the experiment. These are given below in table 1.

Table 1: Measured quantities of the Rüchardt apparatus

Diameter of ball	15.85 ± 0.01 mm
Mass of ball	16.20 ± 0.15 g
Volume per bottle	2.4 ± 0.1 L
Volume of manifold	0.48 ± 0.02 L

Notes on operation

When planning and carrying out your experiment, there are a few things to be aware of:

- **Gas flow** - Gas flow into the system maintains the oscillations against the light damping. The inflow is regulated by a pressure reduction valve on the cylinder and a bleed-cock valve on the Rüchardt manifold. The gas flow should be kept low and adjusted carefully since high flow rates will cause the ball bearing to be ejected from the apparatus, possibly damaging the lights, the ceiling, or the students.

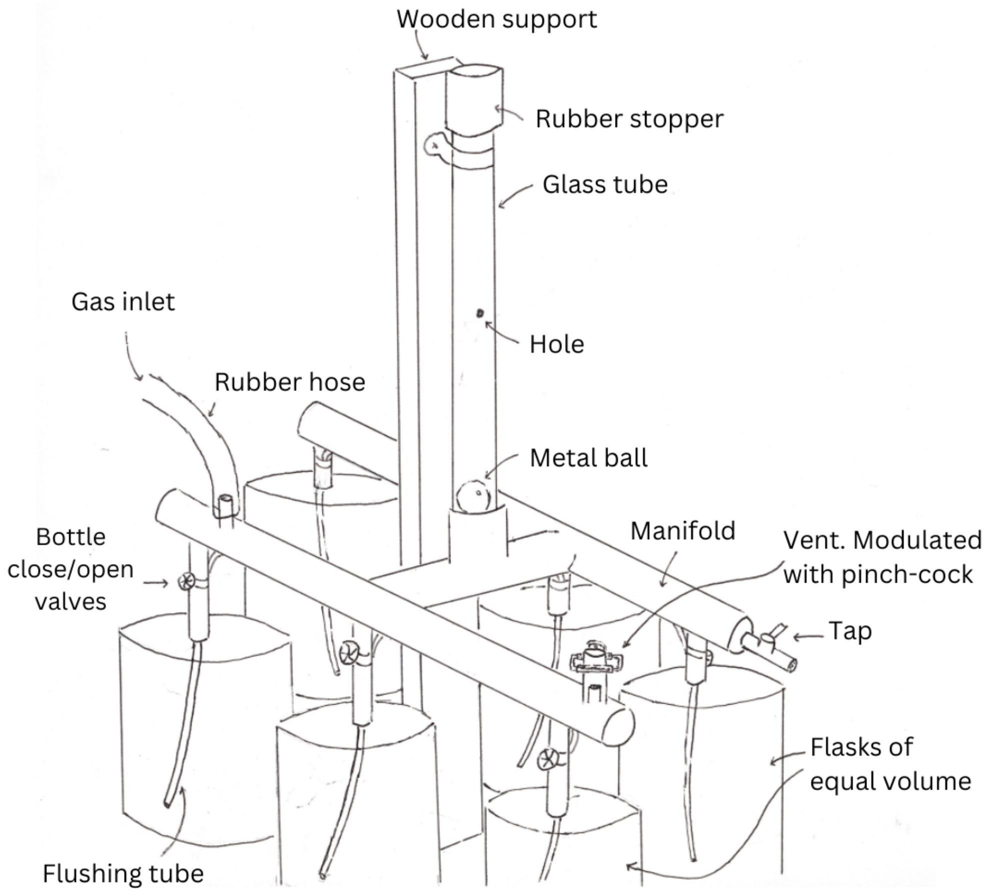


Figure 2: A sketch of the adapted Rüchardt apparatus showing how the gas bottles are connected to the manifold. The number of bottles (and hence the overall volume) is controlled through the individual open/close valves at the top of each bottle. Created by Will Gaffney (second-year lab student, 2024).

- **CO_2 freezing** - CO_2 has a very strong deviation from ideal gas behaviour that causes it to cool upon expansion. This strong cooling can make the valve on the gas cylinder s pressure regulator ice up and flutter, giving irregular gas flow and making it impossible to get accurate measurements of the oscillations. To avoid this, use the smallest possible flow rate of CO_2.
- **Rüchardt gas valves** - correct operation of the gate valves and capillary tubes connecting the flasks to the manifold is essential to allow the proper flushing of the system and then to maintain a fixed volume of gas insise the apparatus. Make sure you familiarise yourself with their operation.

Procedure

With the apparatus described and the relevant physics discussed, it is now time to design and execute the experiment that achieves the Experiment Objectives. Discuss this with your group and your demonstrators.

Task

Your guiding equation for this experiment is equation 8. In order to design an experiment that allows you to accurately measure γ you should consider:

1. Knowing the quantity you wish to measure (γ) and the equation you have derived, what sort of plot or data will you need to carry out this experiment?
2. How you will be able to process your data to obtain the measurement of γ for each gas? What plots will you need to make? What feature of the plot will allow you to get your value of γ for each gas?
3. Which gas should you start with? Is there a preferred order in which you should experiment with each gas? **Note: it is recommended that you flush the flasks with the new gas for at least 15 mins before starting your observations.**
4. How will you determine the atmospheric pressure?
5. What are the errors and uncertainties that will creep in here? Are there ways you can mitigate these?

Write down an outline of your experiment plan and discuss with your demonstrator before commencing.

While carrying out your experiment, remember to include the following in your logbooks

1. Labelled diagrams of your setup.
2. Details of your process including processes which didn't end up working out.
3. Assumed values.
4. Sources of error.
5. Sanity checks to validate your initial observations and any preliminary results. In other words, how do you know you're on the right track before you get to your final results?

Calculations and Discussion

Having accumulated your data, you are now in a position to determine the adiabatic index for the three gases you have analysed.

Task

1. Plot the relationship between oscillation period and volume. Is there any departure from the relationship in Equation 7? If yes, suggest possible reasons.
2. From your plot, determine a value for γ for each gas you have tested.
3. There are values for several gases given in Table I of Koehler 1951. Are your values significantly different? Suggest and discuss possible sources of systematic error. Recall that differences of $\pm 1\sigma$ are expected to occur just due to random experimental variation about 32% of the time, whil $\pm 2\sigma$ differences are expected only 5% of the time.

As you process your results, remember to **propagate your errors and display error bars on key plots.**

References

Koehler, W. F. (Feb. 1951). "A Laboratory Experiment on the Determination of γ for Gases by Self-Sustained Oscillations". In: *American Journal of Physics* 19.2, pp. 113–115. DOI: `10.1119/1.1932723`.

Appendices

A Laboratory Experiment on the Determination of γ for Gases by Self-Sustained Oscillations

W. F. KOEHLER
U. S. Naval Postgraduate School, Annapolis, Maryland

(Received July 3, 1950)

The apparatus used is a Rüchardt Apparatus with a small hole bored through the wall of the tube which permits gas to escape when the ball is above the hole. The ball is caused to oscillate symmetrically with respect to the hole by adjusting an inlet valve. When adjusted, the constant rate of inflow of gas is equal to the average rate of escape of gas. The resulting fluctuation of the pressure is sinusoidal to a high degree of approximation and maintains the oscillations of the ball indefinitely. An elementary theory predicts that γ can be determined from the measurement of the period of oscillation, mean pressure, mass of the ball, diameter of the ball, and volume of the container. Results for most of the common gases are reproducible to within 0.2 percent and differ from the accepted values by 1 percent.

ANY teacher familiar with Rüchardt's method of measuring γ for gases appreciates, from a pedagogical point of view, the value of an experiment using the method. The following is a brief discussion of his apparatus and method of measuring γ. The gas under investigation is enclosed in a container with an attached, vertical, precision-bored, glass tube in which a matching steel ball can move freely. His apparatus is similar to that shown in Fig. 1 without the hole in the wall of the vertical tube. If the ball is given a slight displacement with a magnet it will oscillate. An elementary theory[1] predicts simple harmonic motion with a natural period τ_0 related to γ according to

$$\gamma = 64mV_0/(d^4P_0\tau_0^2). \tag{1}$$

The remaining symbols in Eq. (1) can be identified in Fig. 1. When one performs an experiment to test this elementary theory, he finds that the experimental values of γ differ from the accepted values by as much as 15 percent depending upon the nature of the gas and upon the size and shape of the container. Consequently, many have attempted to improve or modify the method. For example, Rinkel[2] measured the initial drop of the ball and claimed better results than those obtained from a measurement of the period of oscillation. Brodersen,[3] using a photographic apparatus to record the displacement, obtained good results for air. Clark and Katz[4] used two gas containers connected by a cylindrical tube with a matching piston. They drove the piston electrically and collected data for a resonance curve. Their values of γ are probably the best experimental values published to date. However, their apparatus is too elaborate for routine laboratory use. The disagreement between the elementary theory and the experimental results

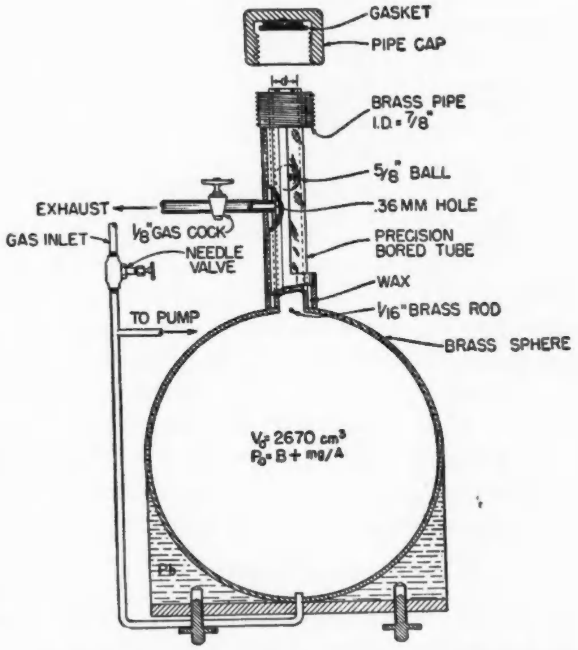


FIG. 1. Apparatus for self-sustained oscillations.

---

[1] M. W. Zemansky, *Heat and Thermodynamics* (McGraw-Hill Book Company, Inc., New York, 1943), pp. 107–110.
[2] R. Rinkel, Physik. Z. **30**, 805 (1929).
[3] P. H. Brodersen, Z. Physik **62**, 180 (1930).
[4] A. L. Clark and L. Katz, Can. J. Research **21A**, 1 (1943).

obtained with the original Rüchardt apparatus is to be expected because the observed oscillation is highly damped which precludes a precise measurement of the period. Furthermore, the measured period of this damped oscillation is not the natural period τ_0 in Eq. (1). Still further, the elementary theory neglects the effect due to heat conduction, the fact that actual gases do not obey the ideal gas law and the fact that the effective mass of the oscillator is not the mass of the ball.

The apparatus herein described is a Rüchardt apparatus with an additional feature to maintain the oscillations. The additional feature is a small hole which is drilled through the wall of the tube as shown in Fig. 1. The ball is caused to oscillate in the tube by adjusting the needle valve so that the ball rises slowly in the tube until it passes the hole at which time a small amount of gas escapes to start the oscillation. The escaping gas causes a drop in pressure which contributes to a periodic driving force which maintains the oscillations. The ball can be made to oscillate symmetrically with respect to the hole by finer adjustment of the needle valve. The oscillations are maintained as long as the gas passes slowly through the apparatus.

TABLE I. Results of determining γ by self-sustained oscillations.

Gas	γ (Experimental)	γ (Accepted)	Percent diff.
CO_2	1.284 ± 0.001	1.300	1.1
NH_3	1.305 ± 0.002	1.315	1.0
O_2	1.382 ± 0.001	1.396	1.0
N_2	1.386 ± 0.001	1.403	1.2
H_2	1.345 ± 0.002	1.405	4.2
A	1.631 ± 0.001	1.670	2.3
He	1.563 ± 0.002	1.667	6.2

A detailed analysis[5] of this oscillator yields

$$\gamma = 64mV_0/(d^4P_0\tau^2)DEGH,$$

where τ is the period of oscillation, $DEGH$ is a product of four correction factors and the other symbols can be identified in Fig. 1. The quantity D corrects for the damping; E corrects for the fact that the mass of the ball is not the effective mass of the oscillator but the mass of the ball increased by a fraction of the mass of the oscillating gas; G corrects for the fact that actual gases do not obey the ideal gas law; and H corrects for the effect of the finite thermal conductivity of the gas which prevents the oscillations from taking place adiabatically. The effects due to the hole in the tube, the volume of the tube and the slow passage of the gas through the apparatus can be ascertained from the theoretical expressions for the correction factors. Briefly, the escaping gas during the upper half of the displacement cycle causes a drop in pressure and the incoming gas causes a rise in pressure. A Fourier analysis of the fluctuation in pressure about the equilibrium value shows that only the first harmonic is necessary to describe the fluctuation in pressure due to the slow passage of gas through the apparatus and that this harmonic slightly leads the displacement of the ball which provides the driving force. The magnitude of this harmonic and its phase relative to the displacement depends upon the thermal conductivity of the gas, area of the hole, size and mass of the ball, ratio of surface area to the volume of the container and the period of oscillation. The apparatus shown in Fig. 1 is designed so that each of the correction factors is approximately unity for the common gases. The design is determined from a consideration of the constants of the apparatus which appear in the theoretical expressions for the correction factors. As a general criterion, the container should have a small surface area to volume ratio, the ball should be large and dense and the hole should be small.

If one assumes that the four correction factors differ from unity by a negligible amount, the equation

$$\gamma = 64mV_0/(d^4P_0\tau^2) = C/(P_0\tau^2) \tag{2}$$

can be derived simply from fundamental principles as for the original Rüchardt apparatus. The constant C for this particular apparatus is 13.00 in. of Hg-sec^2. The results of Table I were obtained by substituting measured values of the equilibrium pressure and period of oscillation in Eq. (2). The accepted values of γ in Table I, except those for ammonia and oxygen, are those values reported by Clark and Katz.[4] These values are chosen for comparison because Clark and Katz measure γ at various values of pressure and compare their extrapolated value at zero pressure with that obtained from spectroscopic data.

---

[5] W. F. Koehler, J. Chem. Phys. 18, 465 (1950).

The accepted values of γ for ammonia and oxygen are taken from Euchen's[6] "Table of Most Probable Values."

The advantages of this method are as follows: (1) The oscillations can be maintained indefinitely and the period can be measured with a precision of 0.1 percent. (2) Various gases can be introduced simply into the apparatus. (3) It provides an excellent exercise for the interpretation of experimental errors. Note that all quantities in Eq. (2) can be conveniently measured to within 0.1 percent and that the final result may differ from the accepted value by several percent. Note also, that if a relative method is used, better agreement with the accepted values can be obtained.

The disadvantages of this method are as follows: (1) The apparatus is somewhat more complicated to construct than the original Rüchardt apparatus. (2) Compared to the results from high frequency methods, this method yields poorer results for gases which have a large thermal conductivity. (Note the results for hydrogen and helium.) This, however, can be said about most laboratory methods using low frequency oscillations. It is due to the fact that the thermal conductivity correction increases with the period of oscillation. (3) This method yields poorer results for gases which deviate considerably from an ideal gas than for gases which approach an ideal gas. However, the G correction factor $-(P/v)(\partial v/\partial P)$ for most of the common gases at average laboratory conditions differs from unity by less than one percent.

The following comments may be helpful during the construction and operation of the apparatus. The precision-bored glass tubes with matching ball bearings (0.001 in. diameter difference) were obtained from the Corning Glass Works for approximately $6.00 each in lots of three. The hole in the wall of the tube was drilled with a wide-angled drill having a carboloy tip. Kerosene was used as a cutting oil during the drilling process which required less than an hour, after a little practice on an ordinary glass tube. If the hole is too large the amplitude of oscillation is large and the motion is not uniform. This can be corrected without introducing appreciable error by closing the end of the exhaust with thin sheet metal in which a smaller hole has been punched with a needle. After a thorough initial cleaning the tube can be kept clean by occasionally pushing a lint-free swab moistened with acetone through the tube. The tube is placed in a vertical position with the aid of a round level placed on the upper end of the tube. When the tube and ball are clean, the tube is in a vertical position and the needle valve is adjusted so that the ball rises slowly in the tube; then, one can observe the ball spinning rapidly. The needle valve used is a Hoke Valve, No. 315. The container used is a brass sphere which was originally made for the top of a flag pole. Somewhat more satisfactory results can be obtained by using a larger container than that shown in Fig. 1 but the additional inconvenience of a larger apparatus does not warrant it.

---

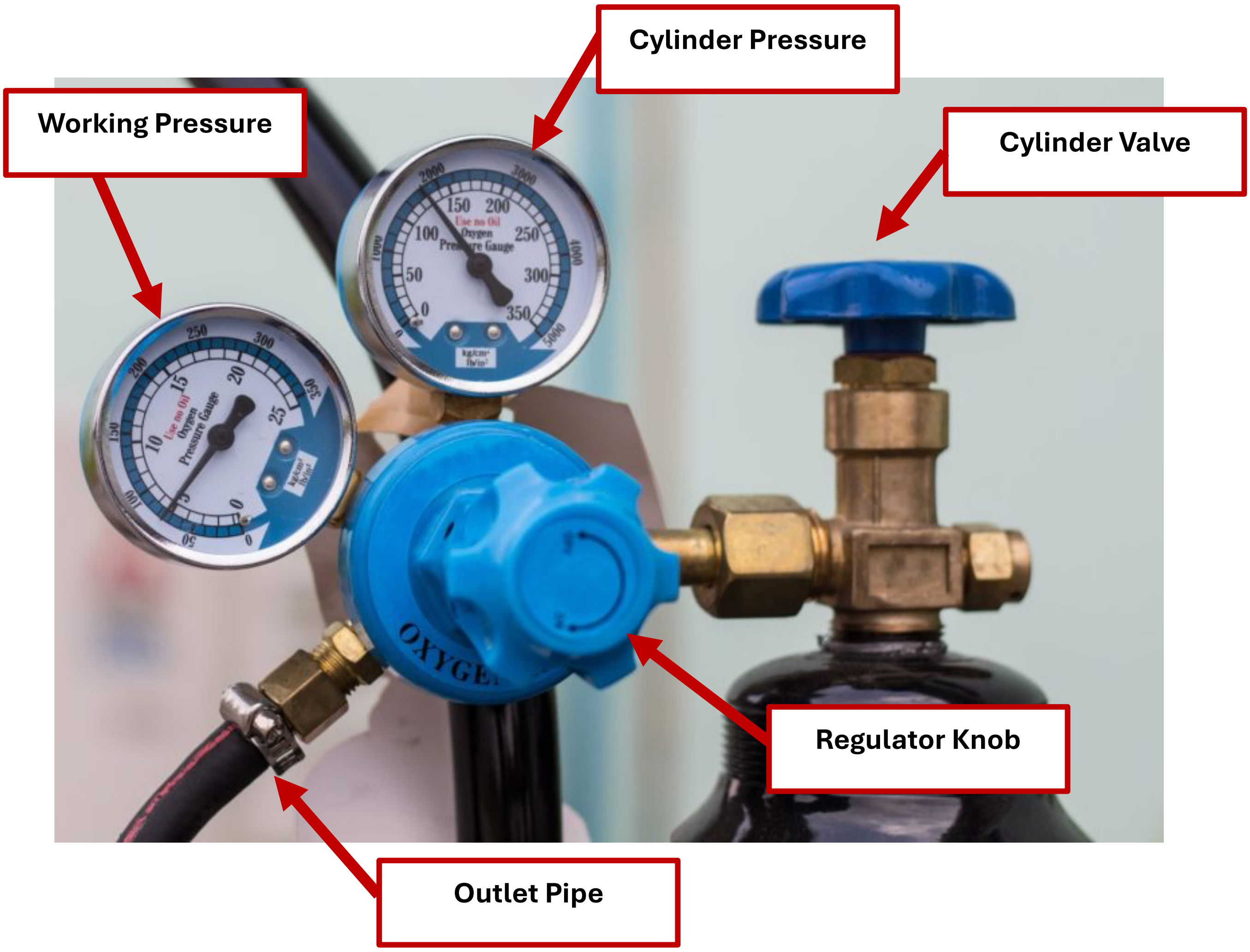
[6] A. Euchen, *Handbuch der Experimentalphysik* **8**, 433 (1929).

Symposium on Molecular Structure and Spectroscopy

A Symposium on Molecular Structure and Spectroscopy will be held at the Department of Physics, The Ohio State University, from June 11 through June 15, 1951. There will be discussions of the interpretation of molecular spectroscopic data as well as methods of obtaining such data. In addition, there will be sessions devoted to those phases of spectroscopy of current interest. A dormitory will be available for those who wish to reside on the campus during the meeting. For further information or for a copy of the program when it becomes available, write to Professor Harald H. Nielsen, Department of Physics, The Ohio State University, Columbus 10, Ohio.

The Symposium will be sponsored this year jointly by the Graduate School and the Department of Physics and Astronomy at The Ohio State University and by the Division of Chemical Physics of the American Physical Society.

Using A Gas Regulator



Opening Gas Flow

1. Check cylinder valve is closed, and regulator knob is turned all the way **out**. Check that any valves on the outlet pipe are closed. There should be no readings on either of the gauges.
2. Open cylinder valve. The cylinder pressure gauge should show a reading.
3. Start to turn the Regulator knob **in** until the working pressure gauge shows a reading. You can now control the gas outflow through the connectors on the outlet pipe or by using the regulator knob.

Closing Gas Flow

1. Turn the cylinder valve off first.
2. Open any valves on the outlet pipe to bleed the gas out of the regulator. The reading on both gauges should drop to zero.
3. Close any outlet valves.
4. Wind the regulator valve all the way **out.**

EQUIPMENT USAGE NOTE

Analogue barometer

UNIVERSITY of TASMANIA

July 6, 2021

The mercury barometer

Summary

Barometers are instruments used to measure atmospheric pressure, and have a rich history of technological development due to the correlation between variations in pressure and changes in weather. The importance of predicting weather, especially during the age of sail cannot be overemphasised, which in great part spurred many developments in our understanding of thermodynamics and its applications, along with clever and robust technologies and instruments to measure quantities of interest. One such example is the mercury barometer, a device for providing an accurate measurement of atmospheric pressure.

The device

This particular barometer is a construction of Haratio Yeates of 97 Collins Street, Hobart and was completed on 05/08/1896. At its core, it is a mercury barometer which is used to measure the local pressure. Due to the age of the device, the units of measurement are antiquated, and to extract an accurate measurement, corrections must applied to the measured value.

How does it function?

Traditional barometers function on the principle of equilibrium: the force exerted on an area of fluid by the atmosphere is balanced by the weight of the same fluid in a restricted vessel. Specifically, a tube sealed at one end with a diameter smaller than that of the reservoir is inserted into the reservoir such that the liquid is free to flow from the reservoir into the tube. If the instrument is initialised with the tube full of liquid, following submersion, a vacuum will be created above the fluid and an the fluid will settle at a height when the weight of the fluid column exerted on the reservoir is equal to that of the pressure exerted by the atmosphere, meaning that a measurement the height of the fluid is a proxy measurement of the pressure.

In order to achieve a more accurate measurement, an additional mechanism to compensate for the displacement of mercury between the reservoir and the tube under different pressures is included. This has the benefit of allowing much smaller liquid reservoirs, but payment comes in the form of an additional measurement: the reference level of the reservoir must be set prior to reading the height of the liquid.

A limitation of barometers comes in the form that whilst we are interested in measuring the pressure, our system will respond to other environmental parameters, notably temperature, and thus in order to collect an accurate measurement of pressure, care must be taken to simultaneously record the temperature and pressure, and apply the appropriate corrections. Moreover, the location of the barometer will influence the measured pressure: the altitude of the device, along with the local gravitation potential will influence the measurement.

Exercise 1

You will likely be aware that barometers typically use mercury as the liquid to produce pressure readings. What is the reason for this?

Solution 1

The pressure differential ΔP between the atmosphere and the vacuum (or near vacuum) at the top of the barometer is related to the difference in height h, the density of the fluid ρ and the acceleration due to gravity g through $\Delta P = \rho g h$. Recast, the height of the fluid column will be given by

$$h = \frac{\Delta P}{\rho g} \tag{1}$$

and thus to have a “small”-sized column, one wants to use the highest density liquid possible, and with a density of $13.6\,\mathrm{g/cm^{-3}}$ mercury is the liquid with the highest density. Unfortunately, mercury is acutely toxic and consequently, mercury is not commonly used in mordern scientific instrumentation (indeed, it is banned in the European Union) and friendlier *heavy fluids* are used, such as Sodium metatungstate, but with a density of only $3.1\,\mathrm{g/cm^{-3}}$, a barometer would need to be over four times the height to measure an equivalent pressure differential. This is still better than water, which would need to be 13.6 times the height!

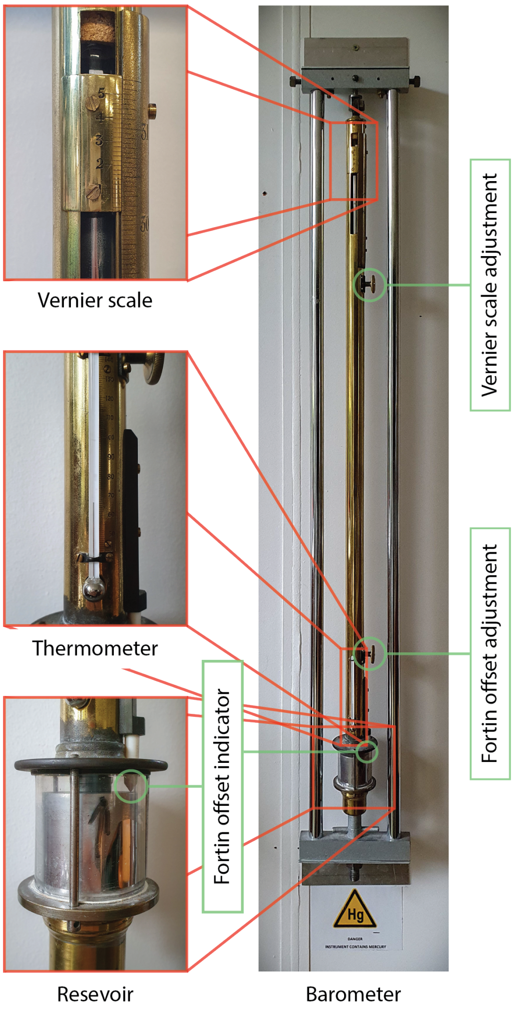


Figure 1: The Fortin barometer with identified components.

The level of care that must be applied when correcting the measured pressure depends on the measurement uncertainty, as there is no point correcting for the local gravitation potential, which alters a measurement by approximately 0.5%, if you have a relative uncertainty of the order of 10%. All correction formulae are provided below, and a detailed discussed of their origins can be found elsewhere [1], along with published tables of corrections [2].

Note that the physics lab has a location described by the latitude and longitude values (-42.90366, 147.32738), and the altitude is estimated as 31.54 m above the mean sea level.

Temperature correction

The correction C_T which corrects for the effects of temperature a T in in Fahrenheit is given by:

$$C_T = P_T - P_m = P_m \left(\frac{1 + L(T - 62)}{1 + M(T - 32)} - 1 \right) \tag{2}$$

where P_T is the temperature corrected reading, P_m is the measured value, and the correction factors are $L = 1.02 \times 10^{-5}\,°\mathrm{F}^{-1}$ and $M = 10.10 \times 10^{-}5\,°\mathrm{F}^{-1}$.

Gravity correction

The correction C_g which corrects for variation is gravity can be approximated at a given angle of latitude φ by

$$C_g = P_l - P_T = P_T \left(\frac{980.616}{980.665} \left(1 - \frac{2637.3 \cos(2\varphi) + 5.9 \cos^2(2\varphi)}{10^6} \right) - 1 \right) \tag{3}$$

and yielding P_l, the local pressure.

Altitude correction

The pressure as measured at altitude can be corrected by calculating the pressure differential ΔP_s:

$$\Delta P_s = P_0 \left(1 - \left(1 - \frac{6.5 \times 10^{-3}}{288.16} H \right)^{5.2561} \right) \tag{4}$$

where $P_0 = 29.921$ inchesHg is the standard pressure at sea level and H is the geopotential height in metres, which can be approximated by the elevation above sea level.

How does one drive it?

The mode of pressure measurement is both formulaic and straightforward:

1. Set the reservoir reference level. Use the knurled knob at the base of the instrument to set the ivory pointer so that the tip is touching the surface of the mercury.
2. Measure the height of the mercury column. Use the knurled knob at the top of the instrument to position the bottom of the Vernier scale in line with the top of the mercury meniscus. Operating with care to avoid parallax error, record the height of the mercury column from the Vernier scale.
3. Record the temperature using the mounted thermometer, use relevant formulae to correct and convert the measurement.

References

[1] LP Harrison. Manual of barometry. *US Dept. of Commerce, Weather Bureau, Washington, DC*, 1963.

[2] Robert J List. Smithsonian meteorological tables. 1968.

Additional resources

- POLUS is a resource for all things related to experimental physics at UTAS

Pressure chapter correction due to temperature

The table 1 is a reproduced version of *TABLE VIII (c)* in the previous barometer documentation, providing the temperature correction to the barometer reading. Note that the correction is to be subtracted in every case.

Temp [° F]	**Pressure [inches Hg]**										
	26.0	**26.5**	**27.0**	**27.5**	**28.0**	**28.5**	**29.0**	**29.5**	**30.0**	**30.5**	**31.0**
30	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004	0.004	0.004
31	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.007	0.007
32	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.009	0.009
33	0.010	0.010	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012
34	0.013	0.013	0.013	0.013	0.014	0.014	0.014	0.014	0.015	0.015	0.015
35	0.015	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017	0.018	0.018
36	0.017	0.018	0.018	0.018	0.019	0.019	0.019	0.020	0.020	0.020	0.021
37	0.020	0.020	0.021	0.021	0.021	0.022	0.022	0.022	0.023	0.023	0.024
38	0.022	0.023	0.023	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.026
39	0.024	0.025	0.025	0.026	0.026	0.027	0.027	0.028	0.028	0.029	0.029
40	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.031	0.032	0.032
41	0.290	0.030	0.030	0.031	0.031	0.032	0.033	0.033	0.034	0.034	0.035
42	0.032	0.032	0.033	0.033	0.034	0.035	0.035	0.036	0.036	0.037	0.038
43	0.034	0.035	0.035	0.036	0.036	0.037	0.038	0.038	0.039	0.040	0.040
44	0.036	0.037	0.038	0.038	0.039	0.040	0.040	0.041	0.042	0.043	0.043
45	0.039	0.039	0.040	0.041	0.042	0.042	0.043	0.044	0.045	0.045	0.046
46	0.041	0.042	0.043	0.043	0.044	0.045	0.046	0.047	0.047	0.048	0.049
47	0.043	0.044	0.045	0.046	0.047	0.048	0.048	0.049	0.050	0.051	0.052
48	0.046	0.047	0.047	0.048	0.049	0.050	0.051	0.052	0.053	0.054	0.054
49	0.048	0.049	0.050	0.051	0.052	0.053	0.054	0.055	0.055	0.056	0.057
50	0.050	0.051	0.052	0.053	0.054	0.055	0.056	0.057	0.058	0.059	0.060
51	0.053	0.054	0.055	0.056	0.057	0.058	0.059	0.060	0.061	0.062	0.063
52	0.055	0.056	0.057	0.058	0.059	0.060	0.061	0.062	0.064	0.065	0.066
53	0.057	0.059	0.060	0.061	0.062	0.063	0.064	0.065	0.066	0.067	0.068
54	0.060	0.061	0.062	0.063	0.064	0.065	0.067	0.068	0.069	0.070	0.071
55	0.062	0.063	0.064	0.065	0.067	0.068	0.069	0.071	0.072	0.073	0.074
56	0.064	0.066	0.067	0.068	0.069	0.070	0.072	0.073	0.074	0.076	0.077
57	0.067	0.068	0.069	0.071	0.072	0.073	0.075	0.076	0.077	0.078	0.080
58	0.069	0.071	0.072	0.073	0.074	0.076	0.077	0.078	0.080	0.081	0.082
59	0.072	0.073	0.074	0.076	0.077	0.078	0.080	0.081	0.083	0.084	0.085
60	0.074	0.075	0.077	0.078	0.080	0.081	0.082	0.084	0.085	0.087	0.088
61	0.076	0.078	0.079	0.080	0.082	0.084	0.085	0.087	0.088	0.090	0.091
62	0.079	0.080	0.082	0.083	0.085	0.086	0.088	0.089	0.091	0.092	0.094
63	0.081	0.083	0.084	0.086	0.087	0.089	0.090	0.092	0.093	0.095	0.096
64	0.083	0.085	0.086	0.088	0.090	0.092	0.093	0.095	0.096	0.097	0.099
65	0.086	0.088	0.089	0.091	0.092	0.094	0.095	0.097	0.099	0.101	0.102
66	0.088	0.090	0.081	0.093	0.095	0.097	0.098	0.100	0.101	0.103	0.105
67	0.090	0.092	0.094	0.096	0.097	0.099	0.101	0.102	0.104	0.106	0.108
68	0.093	0.095	0.096	0.098	0.100	0.102	0.103	0.105	0.107	0.109	0.110
69	0.095	0.097	0.099	0.101	0.102	0.104	0.106	0.108	0.110	0.112	0.113
70	0.097	0.099	0.101	0.103	0.105	0.107	0.109	0.111	0.112	0.114	0.116
71	0.100	0.102	0.103	0.105	0.107	0.109	0.111	0.113	0.115	0.117	0.119
72	0.102	0.104	0.106	0.108	0.110	0.112	0.114	0.116	0.118	0.120	0.122
73	0.104	0.106	0.108	0.110	0.112	0.114	0.116	0.118	0.120	0.122	0.124
74	0.107	0.109	0.111	0.113	0.115	0.117	0.119	0.121	0.123	0.125	0.127
75	0.109	0.111	0.113	0.115	0.117	0.120	0.122	0.124	0.126	0.128	0.130
76	0.111	0.113	0.116	0.118	0.120	0.122	0.124	0.126	0.128	0.131	0.133
77	0.114	0.116	0.118	0.120	0.122	0.125	0.127	0.129	0.131	0.134	0.136
78	0.116	0.118	0.120	0.123	0.125	0.127	0.129	0.132	0.134	0.136	0.138
79	0.118	0.121	0.123	0.125	0.127	0.130	0.132	0.135	0.137	0.139	0.141

Table 1: *Temperature correction to barometers of the Fortin Pattern.* Applicable to readings of Mercury barometers of the Fortin pattern, with brass scales extending from the cistern to the top of the mercury column to reduce them to 32° F.