41511

THIRD-YEAR PHYSICS LABORATORY

MUON LIFETIME MEASUREMENT

Introduction

Muons occur naturally in the laboratory as a weak, generally downward moving flux. They originate in the atmosphere at 20 - 30 km altitude as a secondary product of the interaction between atmospheric nuclei and cosmic radiation (ionised nuclei travelling through interstellar space at highly relativistic velocities). Being charged, muons lose energy, and hence velocity, by a variety of processes leading to ionisation of the ambient medium along their tracks. If slowed sufficiently, they may be captured by an atom to produce a *"muonic atom"*, in which the muon replaces an electron. In this way they may stay within a piece of apparatus for a time long compared with their normal transit time.

Muons are unstable particles and decay spontaneously into an electron or positron together with a neutrino/anti-neutrino pair, i.e.

$$\mu^- \rightarrow e^- + \nu_e + \overline{\nu_e}$$
$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu_e}$$

Muon decay follows the normal radioactive decay law. Thus, if N muons were present at any time t the number δN decaying in time interval δt would be related to N by

$$\delta N = -\lambda N\,\delta t$$

where λ is the appropriate *decay constant.* Integrating gives the usual form met when discussing radioactive decay

$$N = N_0\, e^{-\lambda t}$$

where N_0 is the number of particles present at $t = 0$. Hence we can rewrite our original differential form as

$$\delta N = -\lambda N_0\, e^{-\lambda t}\delta t$$

Here, if $N_0 \equiv 1$, δN would be equivalent to the *probability* of decay. The decay constant λ is related to the *mean lifetime* and the *half life* of the decaying particles, as follows.

$$\tau_{AV} = 1/\lambda \quad \text{and} \quad \tau_{1/2} = 0.693/\lambda$$

This experiment

The aim of this experiment is to deduce λ and hence the *mean lifetime* of the muon from the above equations, using a large number of measurements of the interval between the birth and death of a muonic atom. In the context of this experiment, $t = 0$ is taken arbitrarily as the birth of each muonic atom, while N_0 is the number of atoms observed. The quantity δN is the number of muons decaying within infinitesimal time interval δt centred at time t after the formation of the muonic atom. A suitable rearrangement of the equation $\delta N = -\lambda N_0 e^{-\lambda t}\delta t$ will enable a straight line to be fitted to the an experimental histogram of decay intervals, such that the gradient yields the decay constant λ. The theory behind this rearrangement is as follows.

If we set our equipment to have recording time bins of a fixed with Δt, and we are collecting data in a time bin that extends from t to $t + \Delta t$ after formation of the muonic atom, then the number of recorded decay events, ΔN, will be

$$\begin{aligned} \Delta N &= \int_t^{t+\Delta t} \left(-\lambda N_0 e^{-\lambda t}\right) dt \\ &= \left[N_0 e^{-\lambda t}\right]_t^{t+\Delta t} \\ &= N_0 e^{-\lambda t}\left(e^{-\lambda \Delta t} - 1\right) \end{aligned}$$

If we then take the natural logarithm of both sides, we have

$$ln_e(\Delta N) = ln_e(N_0) - \lambda t + ln_e\left(e^{-\lambda \Delta t} - 1\right)$$

This has the linear form $y = mt + c$, where we identify the slope as $-\lambda$ and the intercept as

$$ln_e(N_0) + ln_e\left(e^{-\lambda \Delta t} - 1\right)$$

Thus a plot of $ln_e(\Delta N)$ versus t gives a slope that we can identify with the decay constant. The mean lifetime of the muon will follow as $\tau_{AV} = 1/\lambda$. .

The experimental equipment

You are provided with a light-tight box containing a large cylindrical block of plastic scintillator with a photomultiplier (PM) tube optically coupled to it. The scintillator emits a light flash whenever a fast charged particle (electron, muon, etc.) passes through it. The photomultiplier converts this light flash into a few photo-electrons at its photo-cathode and these are multiplied through a cascade process involving acceleration in an electric field, impact and re-emission at a number of electrodes, called dynodes, arranged as a series of plane-parallel venetian blinds. The resulting pulse of $\sim 10^6$ electrons is collected at an anode that has its voltage supplied through a resistor from a high voltage source (EHT) which also supplies the intermediate voltages for the dynodes. The form of the resulting pulse at the anode is negative with an exponential tail. This is output to the electronics, is shown in Figure 1, where the type LM710 comparator generates a positive rectangular TTL logic level (0V - 5V) pulse provided the input pulse from the PM tube is greater than the reference voltage, V_{REF}, for the comparator. The amplitude of the input pulse is dependent on the intensity of the light flash and the multiplication factor of the PM tube. The latter is strongly dependent on the EHT voltage.

Several different types of ionising events occur in the scintillator. Muons passing through or stopping in the scintillator leave *hundreds of MeV* energy in it, giving rise to large light flashes and hence large amplitude pulses. Electrons from muon decay will normally lose all their energy in the scintillator and leave *tens of MeV* energy in it, giving rise to slightly smaller light flashes and hence slightly smaller amplitude pulses. Gamma rays from cosmic rays, and decay of radioactive materials in the environment, will contribute a large background of small pulses with energies *less than about 1MeV* when they make photo-electric interactions in the scintillator.

If V_{REF} is held constant and the EHT voltage is increased, the amplitudes of all input pulses will increase. At low EHT voltages only the largest input pulses, which are those from muons, will trigger the comparator. At higher EHT voltages more and more of the gamma-ray induced pulses will trigger the comparator. At large EHT voltages the so-called *dark-current pulses*, produced by spontaneous thermal emission of single electrons from the photocathode, will also trigger the comparator, giving rise to a very high count rate.

The apparatus should be operated at an EHT voltage such that all initial muon-arrival, and subsequent muon-decay-electron pulses, trigger the comparator, but most gamma rays do not. The correct operating voltage is ~ 1400V-1600V, but you should plot the comparator rate versus EHT voltage yourself to determine the optimum voltage required to detect muons and electrons while avoiding gamma rays. The gamma-ray rate is much higher than the muon/electron rate. Look for a *plateau* in the rate versus EHT voltage, but do not exceed 1800V.

The birth and death of the muonic atoms will be registered as two related flashes, one as the muon enters the scintillator and a second as the muon in the muonic atom decays producing a fast electron or positron. These events are likely to occur at the rate of *one or two per minute*, with the flash pairs *separated by microseconds*. They may be identified with patience by viewing the pulses on an oscilloscope. It is the statistics of these time intervals that you will need to record and analyse.

Time-to-height converter

The PM adapter module is designed to generate two pulses, the first to start the operation of the time-to-height converter (THC) and the second to stop it. If the THC start pulse is related to a muon arrival/stopping/capture to form a muonic atom, then the muon-decay will create a PM pulse that stops the THC. In this case the output will be a pulse with an amplitude proportional to the time between the arrival of the muon and its subsequent decay. The THC is connected to a pulse height analyser (PHA) which divides the input-pulse amplitude range (typically 0V - 5V) into a selectable number of pulse height intervals or bins, e.g. 256, 512, 1024. The PHA measures the amplitude of each input pulse and increments the number in the memory bin corresponding to that pulse amplitude. It displays accumulated data in the form of a histogram, referred to as the *pulse-height distribution*, i.e. the number of pulses of each height that have been detected. In our experiment, the heights correspond to the time intervals between start and stop pulses, and the PHA is actually displaying a *frequency-of-occurrence versus time* distribution. If all start pulses arise from such muon arrival/stopping/ capture events, the PHA will display the exponential-decay time distribution for muons.

In practice many muons do not stop and form a muonic atom. These will generate a pulse associated with their passage, but of course there will be no related decay pulse to be detected. Hence the THC may start on a non-stopping muon pulse, and in this case will stop on the next large pulse of any origin (e.g. another muon passing, a muon-decay electron, or even a gamma-ray) thus registering a false output. In fact most outputs will be of this type since the comparator output count rate is ~10 - 100 sec^{-1} whereas the muon-stopping rate is ~1 - 2 min^{-1}. This problem can be avoided by setting the THC to convert only over a short time range, e.g. 0 - 10 μs . Pulses separated by more than this time will give no output, whereas most muons will decay in <10 μs and so will be recorded.

Digital electronics

This generates the start and stop pulses for the THC. The comparator output toggles a 74LS74 flip-flop. The Q output of the flip-flop generates the start pulse via a monostable and a line driver. The $\overline{Q}$ output generates the stop pulse. Because most pulses are from non-stopping single-pulse events, approximately 50% of all genuine muon decay events will be lost. This is due to the THC starting on a single pulse event and then stopping when shortly thereafter a muon stops in the scintillator forming a muonic atom and commencing a genuine muon-decay process. This is an inevitable characteristic of the method used in the experiment, but has no systematic effect on the final frequency versus time distribution, other than to halve the number of events detected. However, it can be inconvenient when testing the system with a double-pulse generator, since the system can by chance start the THC on the second of a double-pulse pair, and stop on the first of the next pair. If this occurs, the pulse generator should be switched off (or to a low amplitude) momentarily and then back on again several times until the correct triggering occurs.

Experimental procedure

1. Establish the best EHT voltage for the system, discussed earlier. V_{REF} should be fixed at -0.5V using a multimeter or oscilloscope. The time-constant selector switch should normally be set on the *shortest* time constant to minimise the pulse-decay time, thereby maximising the chance of detection of two pulses with very small time separation.

2. The system should now be calibrated using a double-pulse generator set up with very short rise and decay times. Set the pulse amplitude comfortably (×2) above the comparator threshold, and using an oscilloscope set the double-pulse separation to a few μs. Now check the start and stop outputs to see that the stop pulse follows the start pulse by the *double-pulse separation time*. If this is not the case, but the separation is equal to the *double-pulse repetition time*, flick the pulse generator on and off a few times until it is triggering correctly. Set the THC to convert in the range 0 - 10 μs or similar, and set the maximum output voltage of the THC to equal the maximum PHA input voltage (5V for most PHA's, but up to ~8V with our current PHA). Now set the PHA to accumulate mode, and check that the numbers in the bin appropriate to the chosen double-pulse separation is incrementing. Vary the double pulse separation and check that accumulation occurs over the whole or most of the PHA range. Use your oscilloscope or some other suitable method to calibrate the time scale on the PHA. Remember that the accuracy of your muon half-life method cannot be better than the accuracy of this time-scale calibration.

3. You are now ready to begin measurements using muons. The rate is very low (~1 - 2 min^{-1}) so you will need to accumulate for at least several days. Because of the limited number of events, you should accumulate into a relatively small number (128 or 256) of bins. The conversion time range should been chosen to cover about five half-lives of the muon; i.e. 0 - 8 μs or 0 - 10 μs .

4. There will be a small, almost time-independent background due to the background pulses. This can be evaluated by repeating the measurements using a long conversion-time range, say 0 - 80 μs, (i.e. up to ~50 muon half-lives). You will find that the time region above

~10 muon half-lives shows no detectable muon contribution, and thus permits the background to be determined. This background must then be scaled to reflect the same accumulation time, and the same time-bin width, Δt, as was used in getting the actual muon data. Having done this scaling, subtract the background from the muon data before you begin your analysis of it.

5. In your analysis you will be expected to *estimate an uncertainty* for your result. If you simply use a linear-least-squares regression package, such as EXCEL, to determine the slope of the best-fit line to a plot of $ln_e(\Delta N)$ versus t , you will get both a result for the slope and an uncertainty in this result. However, the theory behind such linear-least-squares fitting assumes that all the uncertainties are in the Y-axis values, and the X-axis values have negligible uncertainties. Here this is not the case, as the uncertainties in the $ln_e(\Delta N)$ values and the uncertainties in the t values are of comparable significance. Hence is advisable to consider your evaluation as a two-stage process, using

$$\lambda = t_{AV}^{-1} = \frac{d}{dt}[ln_e(\Delta N)] = \frac{\partial[ln_e(\Delta N)]}{\partial(\text{BinNumber})}\frac{\partial(\text{BinNumber})}{\partial t}$$

Since *Bin Number* is a well determined quantity, a plot of $ln_e(\Delta N)$ versus *Bin Number* satisfies the assumption of accurate X-axis values. Similarly a plot of t versus *Bin Number* satisfies this assumption. Thus such linear-least-squares fitting packages will validly give the slope and its uncertainty in both these plots. You therefore will get the result for λ as the slope of the first of these graphs divided by the slope of the second graph. The uncertainty of the λ result will follow from the usual rules for combining uncertainties when you are multiplying or dividing two quantities.

References

In the SciTech library there are many general introductory particle physics texts to which you can refer for some background information on muons. Here are some sources that you may find useful.

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Hughes, I.S., *Elementary Particles*, 3rd ed., Cambridge University Press, 1991

Marshak, R.E., Riazuddin and Ryan, C.P., *Theory of Weak Interactions in Particle Physics*, Volume 24, Wiley, 1969

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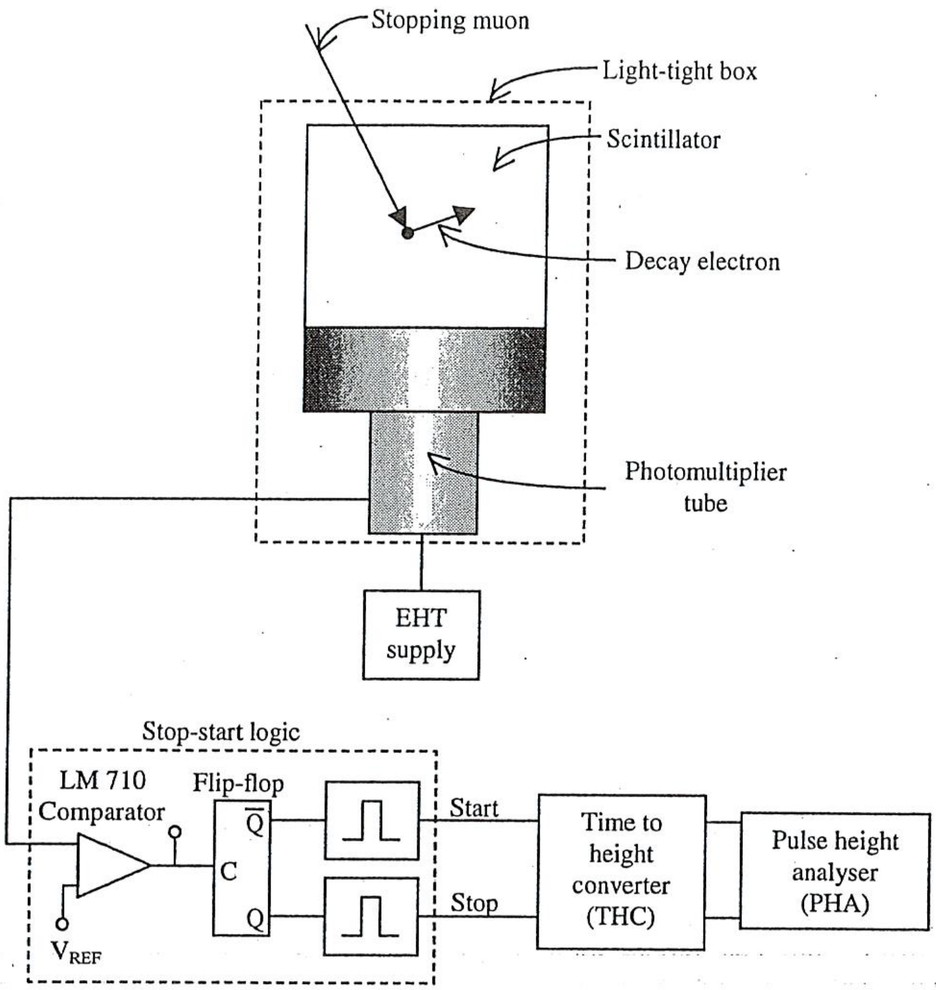
http://www.hep.ph.rhul.ac.uk/muon Website of Royal Holloway College of the University of London

Figure 1: Muon lifetime measurement equipment



Introduction

Muons arriving at the bottom of the atmosphere stop in the plastic scintillator forming muonic atoms – the stopping generates the first signal, and the decay to e^- is a second pulse with Δt a few μsec. Typically these events might be some tens of milliseconds apart, with a small chance of there being a coincidence within a few μsec. The scintillator is a block of plastic in optical contact with a photomultiplier. The photomultiplier involves the photoelectric effect at the photocathode, to liberate electrons, followed by a series of dynodes to multiply the electrons and generate a cascade (by impact and re-emission). The background of counts is associated with spontaneous thermionic emission (single electrons, small pulses); γ rays from radioactive decay in the environment (typically hundreds of eV up to $1\,MeV$); while muons stopping (or passing through without stopping) will deposit hundreds of MeV. Electrons from muon decays will have an energy of a few MeV.

I Setup

The high voltage (HT) on the photomultiplier is set up to accept pulses $> .5V$ (reference level), ie e^- and μ^-, while rejecting thermionic emission and minimising γ counts. The HT (disciminator) voltage must be adjusted to lie just at the 'shoulder' end of the plateau of count rates – beyond the threshold where most pulses are rejected, and before the HT rises so much that many of the γ's are being counted. Typically HT is 1200-1400V but must be graphed (counts using the pulse counter).

II PMA (photomultiplier adaptor)

Use multimeter to check discriminator level (DCV, black to 'level' and red to centre pin on 'pulse' output). The PM tube/assembly output goes to the 'input' of the PMA adaptor unit, and the pulses can be either counted (form the 'pulse' output) or hopefully viewed on the digital CRO screen (although this shouoldbe used wiht the tail pulse generator to get the feel of typical outputs).

III T/H converter, tail pulse generator adjustment

The time/height (T/H) converter looks for two closely spaced pulses and outputs their separation in time as a digital pulse over a certain specifiable range (needs to be $8V$ for our MCA). These pulses can arise from either (i) start pulse from a muon stopping, stop pulse from a muon decay electron; (ii) a transiting muon which does not stop, followed by a lond delay (perhaps 100 ms) until the next event; or (iii) a muon decay electron, followed again by a long delay. Possibilities (ii) and (iii) can be eliminated by selecting the time over which coincidences are counted to be up to 10 μs say. The panel settings have to be taken as amplitude (full range) $8V$, and range (μs) $1V$ equals 1μs (use the multiplier as necessary).

In setting things up the tail pulse generator can be used to create artificial pulses. Settings are pulse frequency, single/double function and delay (for double), $\pm$ pulse selection, rise time (μs), fall time (μs), amplitude, and attenuator $\times 10$, $\times 100$ etc.

IV Count taking

It is necessary to connect the MCA analyser first to known pulses generated by the THC converter, and visibly see the relation between the channel ceing counted in and the set delay time between the pulses. Beware that the pulses being counted may be negative.

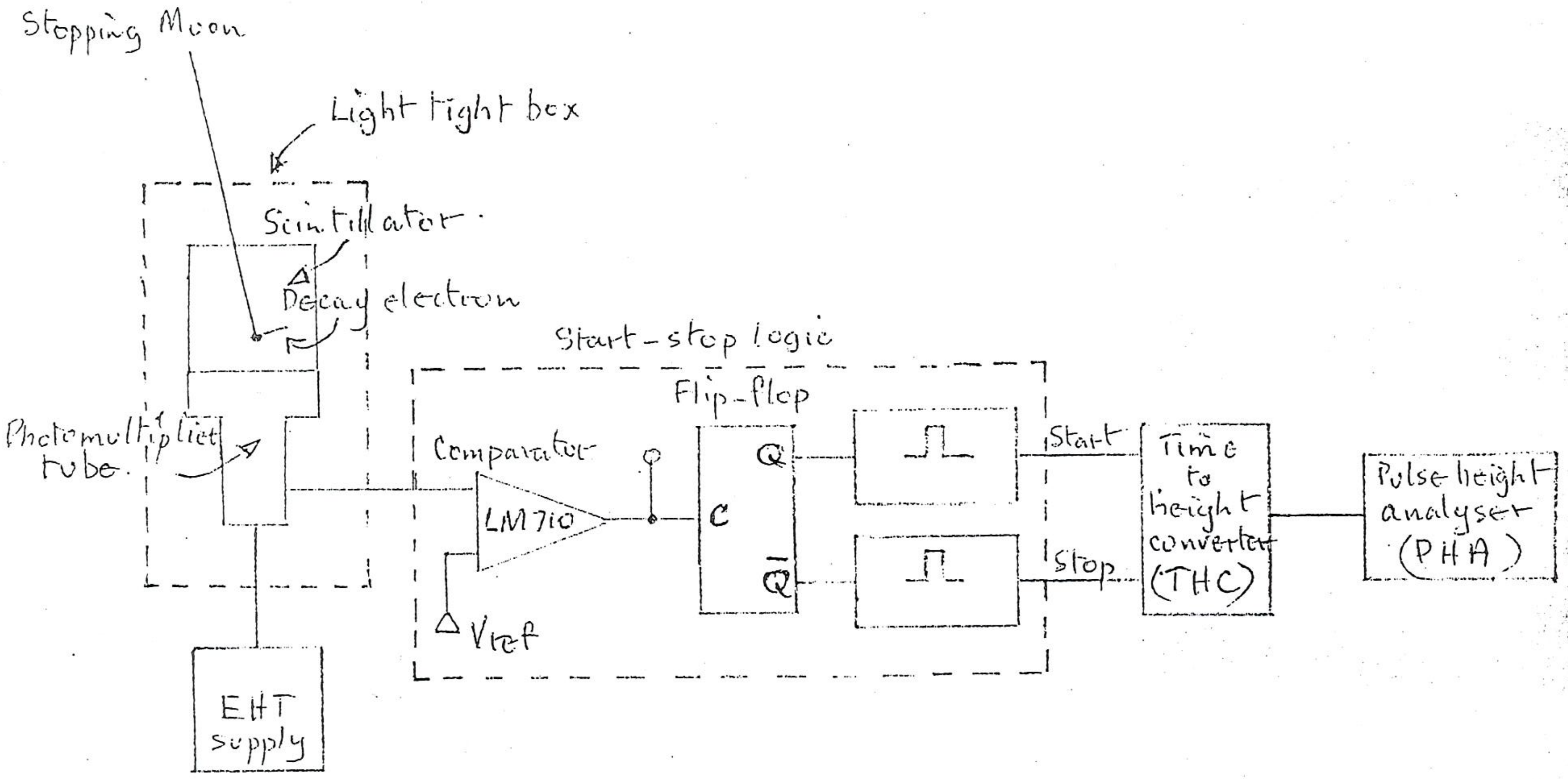


Fig 1: MUON HALF LIFE APPARATUS