KYA 320/321

THIRD-YEAR PHYSICS LABORATORY

Individual experiment:
Solar Spectroscopy

Background Material
And
Prelab Questions

Solar Spectroscopy

Background Material

Caution: This lab requires working at heights (2.8 metres), outdoor work, and work in reduced lighting conditions. One of the calibration lamps when unfiltered can produce light in the ultraviolet spectrum which can result in damage to your eyes if looked at directly for extended periods. You will be using a mirror to reflect the light of the Sun; take care to avoid shining sunlight directly into your eyes or those of your lab partner. Read the accompanying safety manual and be familiar with the emergency procedures and evacuation routes before you begin.

Prelab Questions

This introductory material document contains background information and a few prelab questions; these are to be completed and submitted before you commence the experiment. The information needed to complete the exercises is contained in the experiment background section, your course notes, or in the appendices; however, your own independent research is highly encouraged. Make sure to include references where material has been foraged from elsewhere; this is not only "good form", but making notes of this kind - where to find useful information- is essential should you need to return to the origin of some information.

Context

The experiment demonstrates the use of principles of atomic and statistical physics to answer a quantitative question. The practice of astrophysical spectroscopy dates back to the work of Joseph von Fraunhofer (1814); however, the identification of Fraunhofer's lines as signatures of the presence of terrestrial chemical elements wasn't made until the work of Kirchhoff and Bunsen (1859). Through the late 19th century, it was discovered that the vast majority of stars, including the Sun, show principally absorption line spectra, on top of a continuous emission of light at all visible and near-infrared wavelengths.

Spectra of many stars were obtained, in large part at the Harvard College Observatory, where stars were classified by spectral type according to the strengths of various absorption lines, chief among them neutral and ionised hydrogen, neutral (and rarely, ionised) helium, neutral and singly-ionised iron, calcium, sodium, and magnesium, and, occasionally, very robust molecules like TiO, C_2, or CN. By 1901, Annie Jump Cannon had formalised this approach into the basis of the standard system still used today— all before the physical understanding of the quantum nature of atoms and light.

The Stefan-Boltzmann and Wien displacement laws suggested a link to stellar surface temperature. These laws were eventually explained by Planck (1905) as consequences of the most likely statistical distribution of quantised photon energies for a gas in thermodynamic equilibrium. During this decade, there were strong suspected links between the Harvard

spectral types and temperature, but this was not proven until the 1920s, when quantum mechanics was first applied to the problem of stellar atmospheres and the modern field of stellar spectroscopy began to form.

Every atom and molecule has its own characteristic pattern of spectral lines determined by the energy-level differences between the different states of the species. The presence or absence of lines indicates therefore indicates the chemical composition of a sample. The line energies are quite different for neutral atoms and for ions, and so a quantitative assessment of the concentration of various elements or compounds requires some information about the ionisation state of the sample. This in turn depends on the density and temperature. The link between thermodynamic state of a gas or plasma and the excitation and ionisation states of atoms in equilibrium was first understood by Meghnad Saha (1920). Further, the relative prominence of various lines depends on the quantum mechanical details of the atoms and on the background radiation field around the sample. The first successful application of atomic physics using quantum mechanics to the solar atmosphere was by Cecilia Payne in her 1925 PhD dissertation, which allowed for the measurement of the abundances of chemical elements in stars, as well as probing the temperature, density, and depth structure of stellar atmospheres.

The actual experimental methods to obtain the necessary spectroscopic data developed in tandem with the growing understanding of atoms, light, and atmospheres. The theory made predictions about the strengths of various absorption lines and how they could form under various conditions. In order to test these predictions and measure stellar properties, the light from a source must be *dispersed*, so that different wavelengths fall on a physically different location in a detector that produces an image. The intensity of light on each part of the image can then be measured and transferred either to a graph or a numerical database for analysis.

Stellar spectroscopy has historically challenged the performance specifications of spectrographs and recording devices in ways that spurred technological innovation. The spectra obtained were often required to have sensitivity to intensity variations across wavelength differences of much less than 1 Å (the Ångstrom unit is equal to 0.1 nm $= 10^{-10}$ m). Conversely, to simultaneously measure light from across the whole visible spectrum requires capturing the wavelength range from about 3800-7500 Å. Quantitative measurements require accurate calibration of both the wavelength and intensity scales, independent of instrumental effects or extraneous light sources. Ultraviolet and infrared measurements introduced their own technological requirements. This prompted developments in applied optics, prisms and diffraction gratings. Observations were first recorded on photographic film, and later on solid state, semiconductor-based, digital devices.

Dispersing the Light

In order to be dispersed into a spectrum for measurement, light must pass through (or reflect off of) at least one optical element such as a prism or diffraction grating. Just holding a prism up to the sunlight will produce a low-resolution spectrum, but it will be contaminated by nuisance light from the environment, and will be difficult to interpret because light from multiple physical locations on the disk of the Sun will be directed to different locations on

whatever recording device is used. This muddies the waters because the intensity at a given point could potentially include light of multiple wavelengths from different points on the solar disk.

What we would rather have is a one-to-one relation between a point in the image and a single wavelength, originating from a single point on the Sun. The most common way to achieve this is by building a spectrograph. One of the most straightforward and flexible optical designs is called a Czerny-Turner spectrograph, which involves a narrow *entrance slit*, two concave mirrors (one to collimate and one to focus), and a diffraction grating. In astronomical applications, the light from a telescope set up to track an object is fed into the entrance slit, allowing a high-resolution spectrum to be produced on the detector.

In the case of solar observing, the traditional telescope design is replaced with a "Solar tower telescope". The key innovation of this design is that the telescope and spectrograph do not move to track the Sun, rather it is fed by a *heliostat*. The heliostat does what its name suggests: produce an image of the Sun, and track it so the image remains stable with respect to an experimental apparatus such as a spectrograph or a viewing screen. This is achieved with a system of articulated mirrors: the first mirror tracks the Sun at a rate of 1 revolution per 24 hours, keeping it in the field of view. A second mirror directs the mirror toward the focusing optics and, in our case, one additional mirror steers the beam into the lab.

Safely Creating the Spectrograph Input Beam

Focusing sunlight onto an experiment risks damage to sensitive electronics or optical components, or even fire. The power delivered by sunlight to the ground at the latitude of Hobart could be as much as 1000 W m^{-2} on a clear day in summer. Focusing this light into a narrow point can be extremely hazardous to both equipment and researchers. During winter or periods of cloud this will obviously be less, but cloudy days are not ideal for observing features on the Sun.

One approach to reducing the amount of light is by using a tiny pinhole to image the Sun. This is an effective way to project a small image onto a screen for sunspot viewing. However, the image produced will be tiny, so individual features may be difficult to resolve. The image can be enlarged by moving the screen further from the pinhole, but ultimately the features will be blurred by the diffraction limit of the tiny pinhole. Additionally, it may be discovered by experimentation that moving the screen further away from the pinhole rapidly decreases the brightness of the image.

This decrease in brightness with distance is the key to solving the problem of obtaining a solar spectrum without melting the apparatus. Instead of using a pinhole with a poor diffraction limit and soft focus, a lens with a larger aperture can be employed. In order to avoid burning/blinding hazards, an extremely long focal length is used to spread the photons over a very wide area. This is the classical approach of solar observatories to decreasing the amount of energy per unit area per unit time delivered to the focal plane. These are often oriented vertically or near-vertically to minimise disruption due to air currents— for examples, see the very architecturally interesting Einstein Tower (Potsdam, Germany), the

McMath-Pierce Solar Telescope (Arizona, USA), or a more modern design like the Swedish Solar Telescope (La Palma, Spain— currently the largest optical-wavelength refracting telescope in the world).

In our experiment, a lens of 150 mm aperture and a focal length of 27 m is used to produce an image of the Sun in the optics lab on Level 1 of the physics building. (Technically there are two lenses with slightly different curvature, creating an achromatic doublet that brings blue and red light to the same focus, but the calculations here are the same as for a single lens). In photographic terms, this would be equivalent to an f/180 lens (focal length = 180 times the lens diameter). There are three mirrors involved in the heliostat. Bare aluminium has an average reflectivity of about 90%, but these mirrors are rather old and subject to to high humidity at times, so their reflectivity may be assumed to be about 80% each. In addition, the lens has not been kept in a controlled lab environment for decades, and transmits only about 90% of the incident light. These transmission numbers are very rough estimates only, and are not suitable to calibrate your spectra to absolute intensities (fortunately, you will find a way to work around this limitation).

Prelab Question 1 (Safety Sense Check): You may recall from basic geometric optics that light rays from infinity (such as those originating from the Sun) arrive at a lens parallel to each other and are brought to a focus 1 focal length f behind the lens. Rays arriving at angle θ to the optical axis of the lens arrive at the focal plane at a point that is a distance $h = f\theta$ from the centre of the image. In the following, assume that you are performing the experiment when the solar flux at ground level is near its maximum value for Hobart of (very roughly) 1000 W m^{-2}.

1a. If the angular diameter of the Sun is 32 arcminutes, what is the diameter of the solar image produced in the focal plane?

1b. Given an incident flux of 1000 W m^{-2} and the heliostat/solar tower dimensions and transmittance above, what power per unit area is delivered to the focal plane?

1c. If the light passes through an 0.1 mm spectrograph slit that is 15 mm long in the focal plane, how much energy reaches the next optical element per second? Does this seem like a safe number, if that element is a 1.5 kg ordinary glass mirror?

1d. If the screen at the front of the focal plane has a reflectivity of about 40% (bare wood) or 80% (if covered with paper for sketching sunspots), would this image be safe to look at without eye protection?

If your calculations indicate an unsafe condition, do not proceed with the experiment! Come talk to your demonstrator about obtaining some dimming filters, or examine your calculations more closely.

Absorption Line Formation in a Stellar Atmosphere

The formation of spectral lines in an atmosphere is an enormously complicated subject with 120+ years of development; even now there are some unanswered questions about some types of emission or absorption lines in some contexts. For a complete description of the basics, the text by Gray (2005) is an excellent introduction.

One of the goals of this lab is to attempt to link the strength of an absorption line of singly-ionised calcium (Ca II, also known as Ca^+) to the number of calcium atoms/ions (relative to hydrogen) in the Sun. This takes 4 steps: 1) a definition of the line strength, 2) a corresponding measurement, 3) a physical model of the solar atmosphere allowing you to link line strength to number of absorbing ions, and 4) a statistical correction to account for the Ca ions or atoms that are not in the "right" quantum state to absorb light via the chosen transition.

1. Line strength Defintion

Defining the line strength is relatively straightforward, and standardised. An absorption line represents a decrease in measured flux at the wavelength of the line, formed by the removal of intensity from a ray of light by quantum systems in a specific state as the ray passes through a column of material. If the intensity of light at the chosen wavelength in the absence of the absorption line, called the *continuum level*, would have been I_c, but the actual measured intensity is I_λ, we define the residual intensity $R_\lambda \equiv I_\lambda / I_c$.

Absorption lines have a maximum depth that depends on the quantum physics of the transition itself, but also on the temperature, density, and overall composition of the atmospheric material. In the real world, lines are not infinitely narrow (monochromatic). They may be broadened by a number of processes, including Heisenberg's uncertainty principle (natural broadening), thermal Doppler shifts, bulk motion Doppler shift (convection, turbulence, organised expansion or contraction), and pressure broadening (Stark effect, also known as collisional broadening).

The total line strength is defined by the *equivalent width,* W_λ, which is simply obtained by integrating the flux deficit $D_\lambda = 1 - R_\lambda$ over the entire width of the line. W_λ is numerically equivalent to the width of a line (in Å or nm) that would have $R_\lambda = 0$ over its full extent[1]. Real lines, of course, have much more complicated profiles due to the broadening processes listed above— and that is before we even discuss the impact of *instrumental broadening* due to limited resolution of the spectrograph. One great advantage of defining the line strength by the equivalent width is that W_λ is independent of spectrograph resolution, but this simple measure hides a lot of physics that could be understood by analysis of the *line profiles*, that is, the shape of the R_λ vs. λ curve. **In this lab, we won't require a full calculation of equivalent width,** since we have a shortcut to the calcium abundance (see below).

[1] In other words, the integral of a function gives the "area under the curve".

2. Line Measurement

The strongest lines in the spectrum of the Sun were labelled by Fraunhofer and are listed in Appendix B with their equivalent widths. Because Fraunhofer did not know that spectral lines uniquely identified elements, he simply labelled them alphabetically from long wavelength to short. Note that the strongest lines in the solar spectrum are the lines labelled H and K by Fraunhofer. We now know that these two lines both arise from transitions between the $^2S_{1/2}$ (ground) state of Ca II into two states with spectroscopic terms $^2P_{1/2,3/2}$. This transition involves the change in energy of a single electron[2].

The very broad and deep H and K lines are very easy to detect, which is why the quantitative portion of this lab pays so much attention to them. They are also amenable to a reasonable form of approximate analysis that doesn't require calculating an entire solar model atmosphere or very high precision absolute calibration of the spectrograph. This approach, described in the main section of the lab notes, avoids any complexity associated with attempting to calculate and interpret the equivalent widths. In the following, refer to Figure 1.

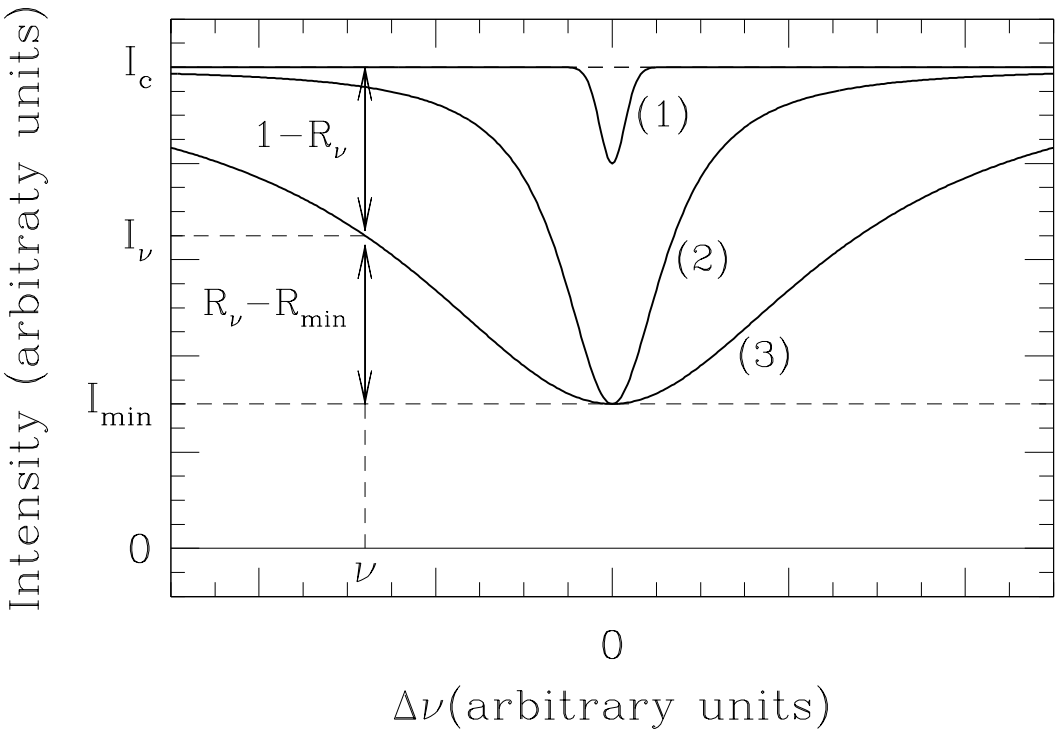

Figure 1: absorption lines of increasing strength with key quantities identified. Idealised absorption lines have their deepest point at the line centre, where the frequency ν is exactly equal to E_{lu}/h, where h is Planck's constant and E_{lu} is the energy difference between the lower and upper energy levels of the absorber. Lines of increasing strength from (1)—(3) are labelled. The continuum intensity I_c, intensity at a given frequency I_ν, and minimum intensity I_{min} are shown. The residual intensity is defined $R_\nu \equiv I_\nu / I_c$.

[2] An intermediate level between these two states is responsible for the infrared calcium triplet, another set of strong lines that were not observed by Fraunhofer because their wavelengths are approximately 8500 Å, beyond the range of visible methods.

The line strength is ultimately a measure of the total amount of light removed from the beam by absorption (or scattering). Determining the absolute calibration of the heliostat, spectrograph, and camera is a tricky procedure, which we will avoid by working with relative intensities. Fitting a straight line to the spectrum for a range of frequencies/wavelengths around the line, but excluding the absorption line itself, will give the continuum level. The depth of the line at any frequency is then just given by the residual intensity R_ν.

The possible values of residual intensity can only range between $0 \leq R_\nu \leq 1$ at each frequency (or wavelength), and this is the quantity you will want to measure and analyse in the lab.

3. The Link between Line Strength and Calcium Abundance

While measuring the strength of the H and K lines is relatively easy, relating this measurement of the equivalent width of the H and K lines to the concentration of calcium atoms is something that you could spend many months on, investing massive effort into coding tasks. What distinguishes the level of effort required is the precision required. Here, we will aim at getting measurements that agree within a factor of a few of the best published values; this can be done quickly with a little bit of understanding of the structure of the solar atmosphere, and some simplifying approximations. The remaining prelab questions will guide you through the justification for these approximations.

Historically, the field began with analyses of strong absorption lines formed under the assumption of *local thermodynamic equilibrium (LTE)*, and the approximation that for relatively thin layers of the solar atmosphere it could be treated in plane parallel geometry, as opposed to spherical. We will adopt both of these assumptions in the analysis of the solar spectrum. The assumption of LTE just means that the ionisation and excitation of atoms as well as the statistical distribution of particle speeds are governed by the kinetic temperature of the gas, but the distribution of photon energies may be given by a different distribution (in this case, the Planck function evaluated at a slightly different temperature).

A key spectroscopic observation is that there are very common patterns that many lines follow, with regard to correlations between the line strength and the profile (or shape) of the line. These patterns are summarised here, noting that there is wide variation in the understanding of line formation in different environments, so only some general outlines are given.

In most lines, the uncertainty principle contributes a very small amount to the line profile. For very weak lines (the line labelled (1), in Figure 1), the typical profile is Gaussian in shape, because they are dominated by Doppler shifts from random thermal motions, with contributions from other motion (turbulence, ordered motion, rotation) considered to be minor. To measure this profile requires very high spectral resolution, that is, the ability to accurately measure small differences in intensity across very narrow frequency (or wavelength) ranges. Such lines are optically thin even in the line core, and any additional absorbers added to the line of sight will add linearly to the strength of the line.

As lines grow stronger, they tend to reach a maximum depth at the line centre (called *saturation*), characterised by thermal emission typical of the temperature of the layer in the star where the line first becomes optically thick (#2, in Figure 1). Note this is usually nowhere close to the true zero level of intensity, because this layer of the star, while higher up in the stellar atmosphere and at lower temperature than the layers doing most of the emission at nearby frequencies, still has T > 0. The level at which saturation occurs is unfortunately a product of many factors, the most important of which is usually the thermal Doppler broadening. As saturation occurs, the equivalent width can only grow very weakly with the increasing abundance of absorbers, and an accurate calibration of the relation between line strength and abundance requires detailed knowledge of the thermodynamic state of the gas/ plasms at every layer from which light could be emitted/absorbed.

If the number of absorbers continues to grow, the saturated line cannot become any deeper at the line core. Eventually the damping wings due to collisions with other atoms and ions become strong enough to be easily observed (#3, in Figure 1). These wings come about from perturbations to the energy levels of the absorbers by the close approach of other atoms or ions, and can be analysed with collision theory (Anderson 1932, Baranger 1958)[3]. Analysis shows that collisional broadening produces a Lorentzian profile, which has the form of a damped resonance. The damping or broadening parameter has to be calculated from the density, temperature, and perturbing potentials of the atoms in the system, but in many cases this is less difficult than accurately measuring the thermal Doppler profile and its variation with depth. Lines that are strong enough to obviously show a profile with damping wings are historically known as *resonance lines*, because of their Lorentzian shape. From an atomic physics perspective, the resonance lines nearly always involve the ground state of the atom, because these are usually the most strongly populated energy levels and the atoms very stronly tend to transition into these levels.

The first attempts to understand solar absorption lines treated the solar atmosphere as being made of two distinct layers: one layer in which all of the continuum radiation was produced, and a thin layer above that called the reversing layer, where all the absorption lines were formed. This simplicity was in some ways required, given that it was developed as early as 1905-06 by Schuster and Schwarzschild (about the same time as the Planck function, and before the Bohr model).

The opacity of material is proportional to the number density of absorbers, the absorption cross section of each absorber, and the path length along the line of sight. This is quantified in the *mass absorption coefficient*, which is a strong function of frequency. A key insight is that the strength of an absorption line depends on the ratio of the opacity of the stellar material due only to the specific absorbers under investigation (ℓ_ν) to the opacity of the atmosphere in the absence of that absorber ($\varkappa_\nu$).

---

[3] see https://phys.libretexts.org/Bookshelves/Astronomy__Cosmology/The_Fundamentals_of_Stellar_Astrophysics_(Collins)/14:_Shape_of_Spectral_Lines/14.05:_Collisional_Broadening for further information (not mandatory, but conceptually useful).

Table 1:

Quantity	Symbol	Dimensions	Value at λ = 3933.7 Å
Continuum Opacity	$\varkappa_\nu$	cm^2 g^{-1}	0.28
Line Opacity	ℓ_ν	cm^2 g^{-1}	Derived from η_ν plus theory
Opacity Ratio	$\eta_\nu \equiv \ell_\nu / \varkappa_\nu$	—	Derived from your measure of R_ν
Frequency at Line Centre	ν	s^{-1} or Hz	7.62×10^{14}

While determining the opacities is subject to systematic errors, these are correlated and so tend to cancel out when taking the ratio of the two opacities. In this case the ratio of two quantities is better-determined than the value of either quantity on its own. The key variables identified here are defined in Table 1.

Later on, the crude, two-layer approach was improved by Milne and Eddington, who developed their methods independently of each other in the 1920s by assuming that the atmosphere was continuously variable rather than discretely layered, but that the variation was limited to linear change with depth. The Milne-Eddington approach is still used to get rough results (as good as 10% accuracy in some cases), but also in the case of exoplanet atmospheres as a first approximation when only low signal-to-noise data is available.

In the Milne-Eddington approach, it can be shown that *if* an absorption line is produced while the atmosphere is in LTE, *and* the line is purely due to absorption— that is, there is no contribution of scattering to the depth— there is a very simple relationship between the residual intensity R_ν and the relative strength of the continuum absorption, namely:

$$\eta_\nu = \frac{1 - R_\nu}{R_\nu - R_{\rm min}} \tag{1}$$

Refer to Figure 1 for the practical illustration of how to read the quantities in Equation 1 off of a spectrum. The complete derivation of Equation 1 is given in many references, among them Hubeny & Mihalas (2014) ``Theory of Stellar Atmospheres''[4].

The importance of Equation 1 is that it allows you to take two very simply measured quantities and from them calculate the relative opacity of the line absorption compared to the absorption that would have occurred if the specific absorbers were not present, as a function of frequency. Once $\eta_\nu \equiv \ell_\nu / \varkappa_\nu$ has been determined, the number of absorbers is given by the fundamental relation $\ell_\nu = n_{\rm abs}\, \sigma_\nu$. Here, $n_{\rm abs}$ is the number of absorbers per unit mass of material, and σ_ν is the cross section for absorption at frequency ν in cm^2 per absorber. The latter quantity must be computed from atomic and kinetic theory. $\varkappa_\nu$ must be determined by a full stellar atmosphere integration, but here we will use a well-accepted value from the literature, $\varkappa_\nu = 0.28$ cm^2 gm^{-1} (Table 1).

[4] A good introduction is at https://phys.libretexts.org/Bookshelves/Astronomy__Cosmology/The_Fundamentals_of_Stellar_Astrophysics_(Collins)/13:_Formation_of_Spectral_Lines

At this stage, we would ordinarily need to do a lengthy computational analysis of the atmosphere to convert from line strength to an abundance of calcium. This would be achieved either by calculating a stellar model atmosphere from first principles, or by pattern matching to templates from an empirical spectral library. Here will take a short cut to obtain an approximate result with a fraction of the effort.

Because the Ca H and K lines are extremely strong, we can ignore the full complexity of the line profile and deal only with the damping wings. Weak and medium strength lines depend on the thermal Doppler broadening and many other parameters, and show a compound profile that is the convolution of a Gaussian with a Lorentzian (known as a Voigt profile). For very strong resonance lines, we can ignore this complication and deal only with the damping wings, a pure Lorentzian. In this approximation, the total strength of the line can be estimated from a measure of at a single frequency far away from the line centre, so the Doppler broadening doesn't matter.

The relevant calculation is that the mass absorption coefficient (line opacity) at a given frequency is equal to the cross section per absorber times the number of absorbers. The cross section is the product of three factors:

$$\sigma_\nu = \sigma_{\rm cl} f \phi_\nu$$

$\sigma_{\rm cl}$ is the classical cross section for absorption of an electron by an ion, treated as a simple harmonic oscillator. From the classical theory of electromagnetism, this works out to be

$$\sigma_{\rm cl} = \frac{\pi e^2}{m_e c}$$

f is a dimensionless quantum mechanical correction to the classical cross section that accounts for the difference between the quantum and classical calculations. For the strongest permitted lines, f approaches unity. In weaker lines, f may be orders of magnitude smaller.

ϕ_ν represents the line profile function, that is, a mathematical description of how the absorption strength varies with freqency across the line, moving from the line centre at ν_0 to nearby frequencies $\nu = \nu_0 \pm \Delta\nu$. In the general case, ϕ_ν must be determnined by a numerical integration of the Voigt function. For the very strongest lines, like the Ca II H & K lines, we can try to approximate this by a pure Lorentzian, with

$$\phi_\nu \approx \frac{\Gamma}{4\pi^2(\Delta\nu)^2}$$

Γ is the Lorentzian damping parameter, which is determined by two factors: the natural broadening of the line due to the Heisenberg uncertainy principle ($\Gamma_{\rm rad}$) and the pressure broadening of the line due to collisions with other atoms or ions ($\Gamma_{\rm coll}$). The total damping parameter is just the sum of the individual contributions, $\Gamma = \Gamma_{\rm coll} + \Gamma_{\rm rad}$. Because the Ca H & K lines are so strong, they have very large natural broadening. To a very good approximation, $\Gamma_{\rm rad}$ is equal to the inverse of the average lifetime of the upper level, which for Ca II K is 6.6 nanoseconds.

Table 2:

Quantity	Symbol	Units	Value
Classical Cross Section	σ_{cl}	cm² Hz	2.654x10^{-2}
Oscillator Strength	f	—	0.627 for Ca II K
Radiative Damping	Γ_{rad}	Hz	1.51x10^{8} for Ca II K upper level

The relevant values needed to calculate the number of calcium absorbers in the lower level of the K line are given in Table 2. You will estimate the collisional broadening Γ_{coll} and verify that the thermal Doppler broadening can be neglected in Prelab question 2.

Prelab Question 2: We have a shortcut way to get from a measurement of the residual intensity to the number of Ca ions in the ground state of the K line at 3933.7 Å, but there wre certain conditions on the line strength for this shortcut to work. The main condition is that the line is so strongly broadened by the combination of natural and collisional broadening that the thermal Doppler broadening may be ignored.

2a. In LTE, the thermal Doppler broadening $\Delta\nu_D$ may be calculated from the classical Doppler shift formula and the 1-dimensional Maxwell-Boltzmann speed distribution for the velocity component along the line of sight. For a line centred at frequency ν_0, the thermal Doppler width is given by

$$\Delta\nu_D = \frac{\sqrt{2k_BT/m}}{c}\nu_0 .$$

Calculate the thermal Dopper width for the Ca II K line for a temperature of 6000 K and compare this to the frequency difference between the line centre and and at a wavelength shifted from the line centre by ±4 nm.

2b. The total damping parameter is given by $\Gamma = \Gamma_{coll} + \Gamma_{rad}$. From laboratory measurements of the lifetime of the upper level of the Ca K line, we know Γ_{rad} (Table 2). Γ_{coll} depends on the relative speed of Ca II ions and their main collisional partners, mostly neutral hydrogen atoms, and their cross section for collisions. From kinetic theory, the collision rate $\Gamma_{coll} = \langle v_{rel}\rangle / l$, where l is the mean free path. Calculate Γ_{coll} if:

i. The mean relative velocity for two particles with reduced mass m_{red} is

$$\langle v_{rel}\rangle = \left(\frac{8k_BT}{\pi m_{red}}\right)^{1/2}$$

ii. The mean free path for collisions with targets of concentration n is $l = (\pi r^2 n)^{-1}$. The appropriate radius to use for Stark effect perturbations is much larger than the size of electron orbits; here it should be on the order of $20a_0$. The number density of hydrogen atoms in the upper photosphere and lower chromosphere is highly variable, but 10^{22} m^{-3} may be taken as a reasonable average value. a_0 is the Bohr radius.

Combining all of the above, we will find n_{abs}, the number of absorbers per mass of solar material, from your measurement of the line ratio η_ν. Using the relation $\eta_\nu = \sigma_\nu n_{abs} / \varkappa_\nu$, we can substitute in for $\sigma_\nu = \sigma_{cl} f \phi_\nu$ (assuming it is true that the thermal Doppler width can be neglected and the line is very strong) to find:

$$n_{abs} = \frac{\eta_\nu \kappa_\nu 4\pi^2 (\Delta\nu)^2}{\sigma_{cl} f \Gamma}$$

Caution: this formula cannot be used in the saturated core of the line, where the Doppler shift has an impact on the line profile. In the lab, you will measure η_ν at one or more wavelengths, with the corresponding values of $\Delta\nu$.

By this stage of the lab, you will have an estimate for the number of Ca II ions in the ground state of the K line per unit mass of solar material.

4. Statistical correction for the total number of calcium atoms

You are now very nearly ready to give your estimate of the total number calcium atoms relative to hydrogen. Only the calcium atoms that happen to be singly ionised *and* in the ground state of the ion so they can absorb a photon of 3933.7 Å have been measured.

Saha Ionization Equilibrium: In LTE, the relative number of atoms in each ionization state is given by the Saha ionization formula. This links the relative number of atoms in each ionization state in turn by considering the partition function of the atoms, ions and electrons and the thermal energy relative to the ionization energy.

In units appropriate to the solar atmosphere problem (cgs for pressures, eV for energies), the Saha formula is

$$\log\left(\frac{N_{i+1}}{N_i}\right) = -\log P_e - \frac{5040}{T}\chi_i + 2.5\log T + \log\left(\frac{2U_{i+1}}{U_i}\right) - 0.48$$

In this notation, the number of ions in the (i+1)th ionization state relative to the ith ionization state is given in terms of the partial pressure of electrons P_e, the temperature, the ionization energy χ_I necessary to go from i to $i+1$, and the partition functions U of each ion. The partition function is a temperature-dependent statistical mechanical formula describing the number of different accessible states of a system with identical energy. All the logarithms are given to base 10. Appropriate values for the parameters are given in Table 3.

Boltzman Excitation Equilibrium: Still assuming LTE, the relative number of atoms or ions in each electronic energy level is given by the Boltzmann formula. Because no additional free electrons are created or destroyed during an excitation process, this formula directly relates the number in an individual level to the *total* number of ions.

The Boltzman formula is $\frac{N_j}{\sum N_j} = \frac{g}{U} e^{-\chi_j / k_B T}$, where in this case the excitation energy $\chi_1 = 0$ because the lower level of the transition is the lowest energy state of the Ca II ion.

Table 3:

Quantity	Symbol	Units	Value
Electron pressure	P_e	dyn cm^{-2}	≈ 12.6 in the upper photosphere
Kinetic Temperature	T	K	≈ 5700 K where the line forms
Ionization Potentials	χ_i	eV	For Ca: χ_0 = 6.11 eV, χ_1 = 11.87 eV
Partition Function	U	—	$U_0 \approx 1.4$, $U_1 \approx 2.3$, $U_2 \approx 1.0$
Statistical Level Weight	g	—	For j = 1 in Ca II, g = 2.

There wouldn't be much point in trying to derive an estimate for the total number of atoms if we happened to be measuring a line that only involved a few percent or less of the total, because the correction, and its associated uncertainties, would outweigh the measurement.

Prelab Question 3: Use the data in Table 3 and the Saha and Boltzmann formula to find the relative number of neutral, singly-ionised, and doubly-ionised calcium, and the fraction of Ca II ions that are in the ground state of the ion. Give a formula that will allow you to convert from the number of absorbers n_{abs} to the *total* number of calcium nuclei per unit mass. Confirm that deriving the total calcium abundance from the K line strength is a sensible thing to attempt.

Appendix A: Strong Lines in the Solar Spectrum

Table A1. The Strongest Lines in the Solar Spectrum

λ (Å)	Element	W (Å)	Name	λ (Å)	Element	W (Å)	Name
3581.21	Fe I	2.14	N	4920.51	Fe I	0.43	
3719.95	Fe I	1.66		4957.61	Fe I	0.45	c
3734.87	Fe I	3.03	M	5167.33	Mg I	0.65	b_4
3749.50	Fe I	1.91		5172.70	Mg I	1.26	b_2
3758.24	Fe I	1.65		5183.62	Mg I	1.58	b_1
3770.63	H_{11}	1.86		5232.95	Fe I	0.35	
3797.90	H_{10}	3.46		5269.55	Fe I	0.41	E
3820.44	Fe I	1.71	L	5324.19	Fe I	0.32	
3825.89	Fe I	1.52		5328.05	Fe I	0.38	
3832.31	Mg I	1.68		5528.42	Mg I	0.29	
3835.39	H_9	2.36		5889.97	Na I	0.63	D_2
3838.40	Mg I	1.92		5895.94	Na I	0.56	D_1
3859.92	Fe I	1.55		6122.23	Ca I	0.22	
3889.05	H_8	2.35		6162.18	Ca I	0.22	
3933.68	Ca II	20.25	K	6562.81	H_α	4.02	C
3968.49	Ca II	15.47	H	6867~	O_2	*Telluric*	B
4045.82	Fe I	1.17		7595~	O_2	*Telluric*	A
4101.75	H_δ	3.13	h	8194.84	Na I	0.30	
4226.74	Ca I	1.48	g	8498.06	Ca II	1.46	
4310 ±10	*Blend*	7.20	G	8542.14	Ca II	3.67	
4340.48	H_γ	2.86	f	8662.17	Ca II	2.60	
4383.56	Fe I	1.01	e	8688.64	Fe I	0.27	
4861.34	H_β	3.68	F	8736.04	Mg I	0.29	

The information above is taken from Moore, Minnaert & Houtgast (1966, U.S. National Bureau of Standards Monograph 61 "The solar spectrum 2935 Å to 8770 Å"). Only a selection of the strongest lines are given. The equivalent widths are approximate and should be used only as a guideline. The line names refer to the designations applied by Fraunhofer.